Diffusion radar par des surfaces rugueuses aléatoires - Application aux surfaces de chaussée

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OUTLINE

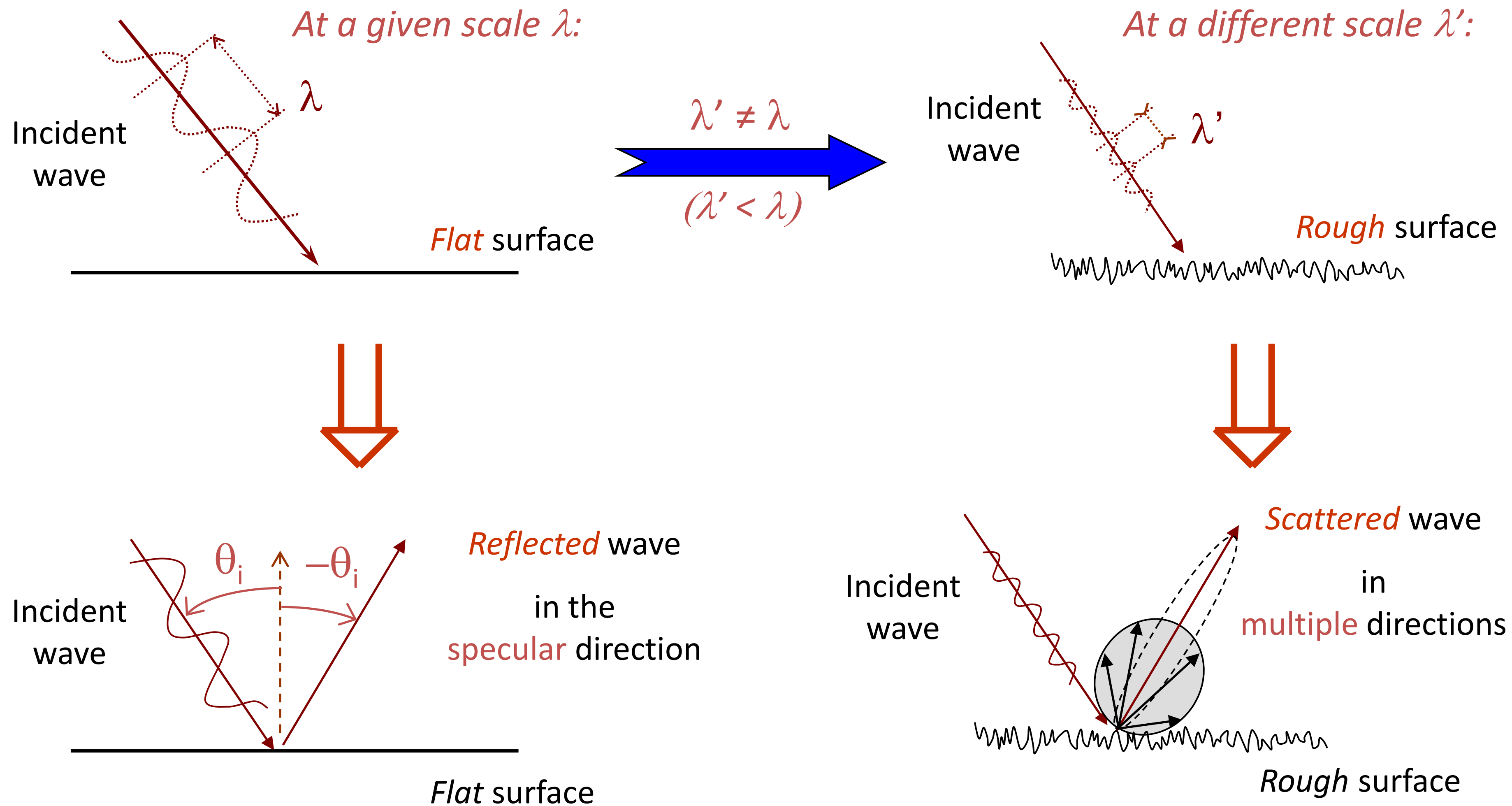
I. **Generalities**
   1. **Rough interfaces: Statistical description**
   2. EM scattering by an interface

II. EM scattering from random rough surfaces: Asymptotic models

III. Applications to GPR

Rough interfaces

Rough interfaces (EM): No surface is perfectly flat at *all scales of EM wavelength*:



Rough interfaces

Random rough surfaces: Different types of variations: $z = \zeta(\cdots)$:

- *Space* variations only: $z = \zeta(x, y)$:
  - agricultural surfaces (ploughed fields, …)
  - surfaces of mountains, sand dunes; ice, …
  - road surfaces, wall surfaces, … (@ high radar frequencies)
  - optical surfaces (non-grounded glasses, …)
  - …
- *Space and time* variations: $z = \zeta(x, y; t)$:
  - sea surfaces
  - surfaces of sand dunes; ice, … (! – long-time observation)
  - …

Random rough surface characterised by:

- p_h: height probability density function (PDF)
- W_h: height autocorrelation function (ACF)

Rough interfaces: Statistical description

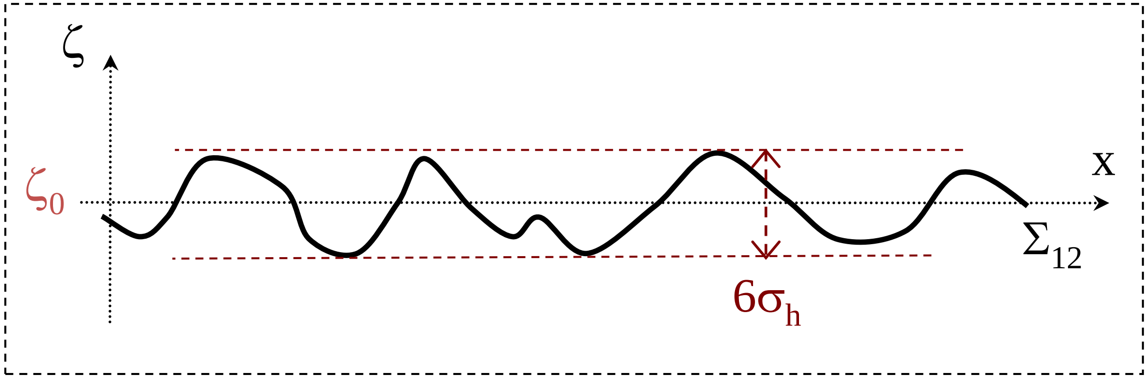
Height PDF p_h:

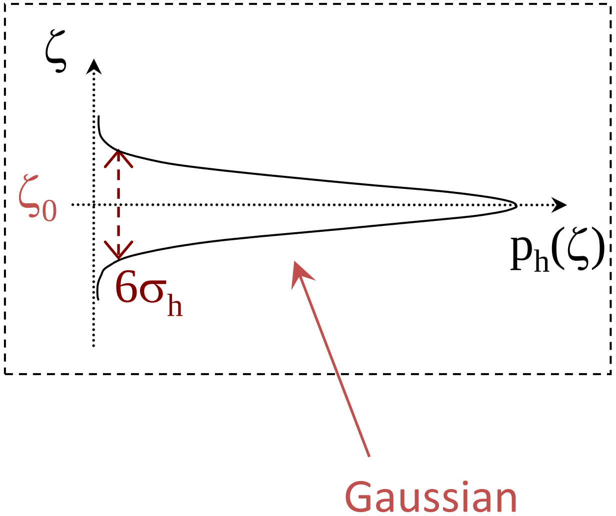
- Mean value $\zeta_0 = \langle \zeta(x) \rangle$
- *Characteristic dispersion around ζ_0*: standard deviation σ_h

Typically → centred Gaussian process (zero mean ζ_0=0):

$$p_h(\zeta) = \frac{1}{\sigma_h \sqrt{2\pi}} \exp\left(-\frac{\zeta^2}{2{\sigma_h}^2}\right)$$

2D profile (1D surface):





Rough interfaces: Statistical description

Height ACF W_h: $W_h = \langle \zeta(x_1, y_1)\, \zeta(x_2, y_2) \rangle$

- Standard deviation σ_h
- Correlation lengths $L_{c,x}$, $L_{c,y}$

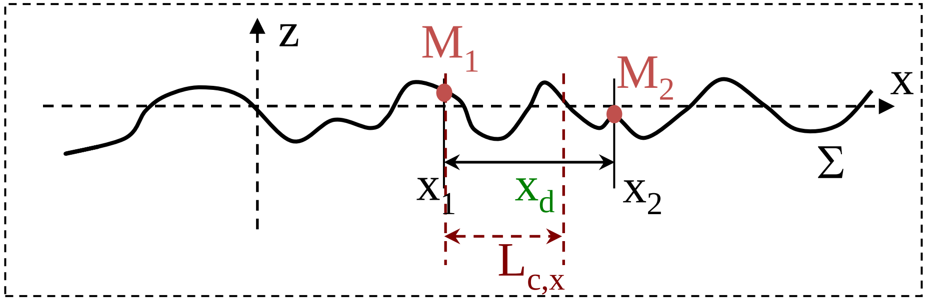
For a stationary process:

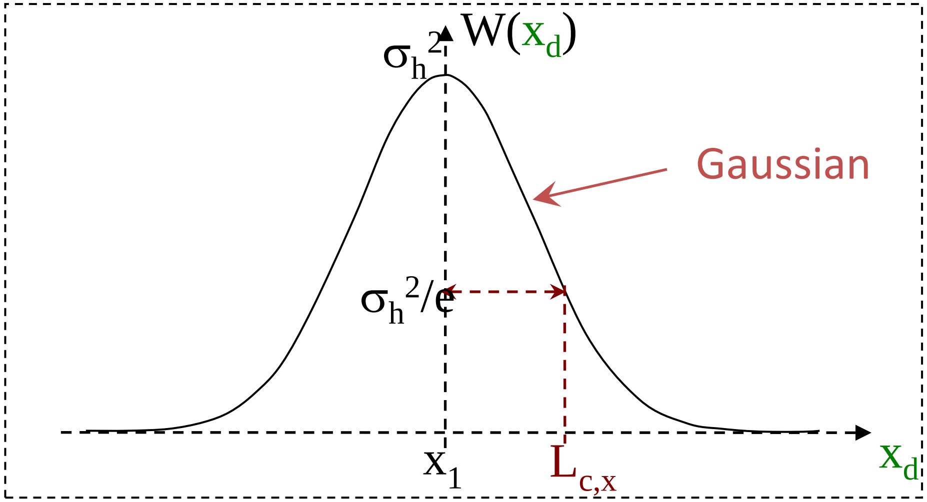
$$W_h = \langle \zeta(\boldsymbol{r}_1)\, \zeta(\boldsymbol{r}_2) \rangle = \langle \zeta(\boldsymbol{r}_1)\, \zeta(\boldsymbol{r}_1 + \boldsymbol{r}) \rangle,$$

with $\boldsymbol{r} = \boldsymbol{r}_2(x_2, y_2) - \boldsymbol{r}_1(x_1, y_1)$

$x_d \gg L_{c,x} \Rightarrow M_1, M_2$ uncorrelated

2D profile (1D surface):





$\Rightarrow$ Gaussian process fully characterized by:

- Height PDF p_h
- Height ACF W_h

Rough interfaces: Statistical description

Height ACFs W_h and spectra S_h: for 1D surfaces $(x, y) \to x$:

- Gaussian, Lorentzian and exponential ACFs:
  1. $W_h(x) = {\sigma_h}^2 \exp\left(-\frac{x^2}{{L_c}^2}\right)$,
  2. $W_h(x) = \frac{{\sigma_h}^2}{1+x^2/{L_c}^2}$,
  3. $W_h(x) = {\sigma_h}^2 \exp\left(-\frac{|x|}{L_c}\right)$
- By Fourier transform (FT), their corresponding spectrum are:
  1. $S_h(k) = \sqrt{\pi}{\sigma_h}^2 L_c \exp\left(-\frac{{L_c}^2 k^2}{4}\right)$,
  2. $S_h(k) = \pi {\sigma_h}^2 L_c \exp(-L_c|k|)$,
  3. $S_h(k) = \frac{2{\sigma_h}^2 L_c}{1+{L_c}^2 k^2}$

→ the FT of a Gaussian is a Gaussian,
whereas the FT of a Lorentzian is an exponential and vice-versa

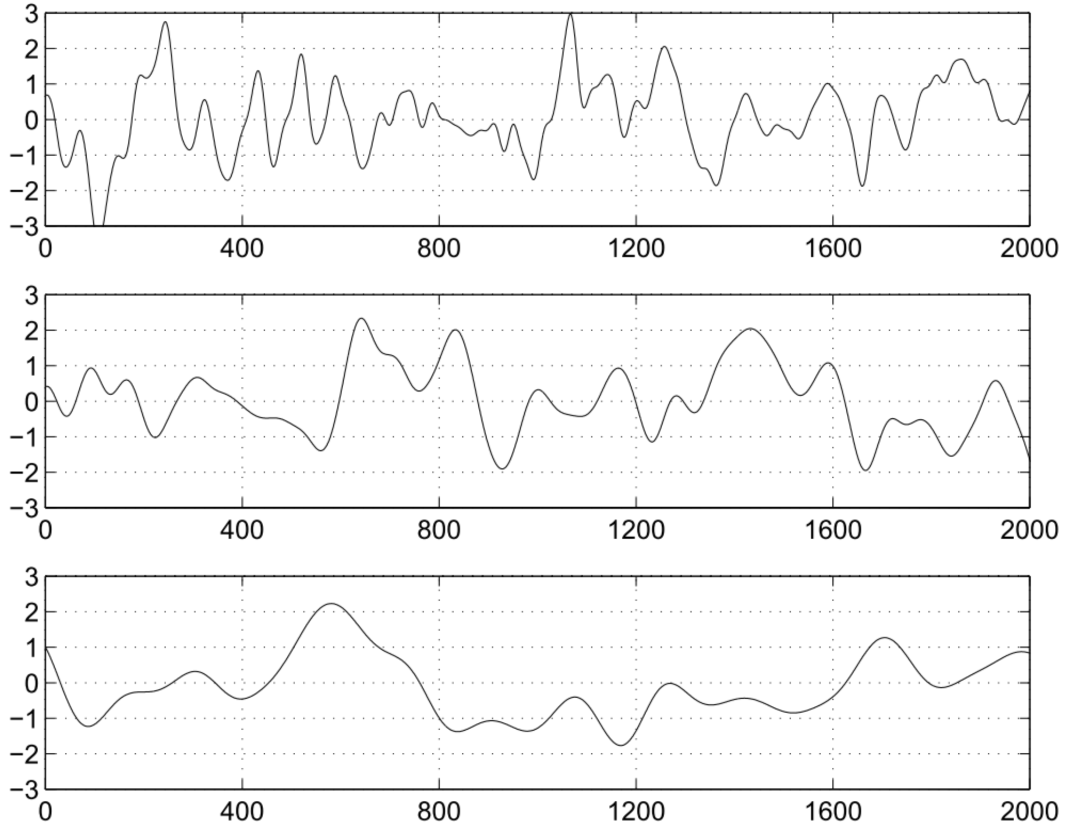
Rough interfaces: Statistical description

Generated (Gaussian) surface – Influence of correlation length L_c:

L_c = 25 m:

L_c = 50 m:

L_c = 100 m:



constant RMS height: σ_h = 1 m

RMS slope: $\sigma_s = \frac{\sqrt{2}\sigma_h}{L_c}$ → $L_c \nearrow \Rightarrow \sigma_s \searrow$

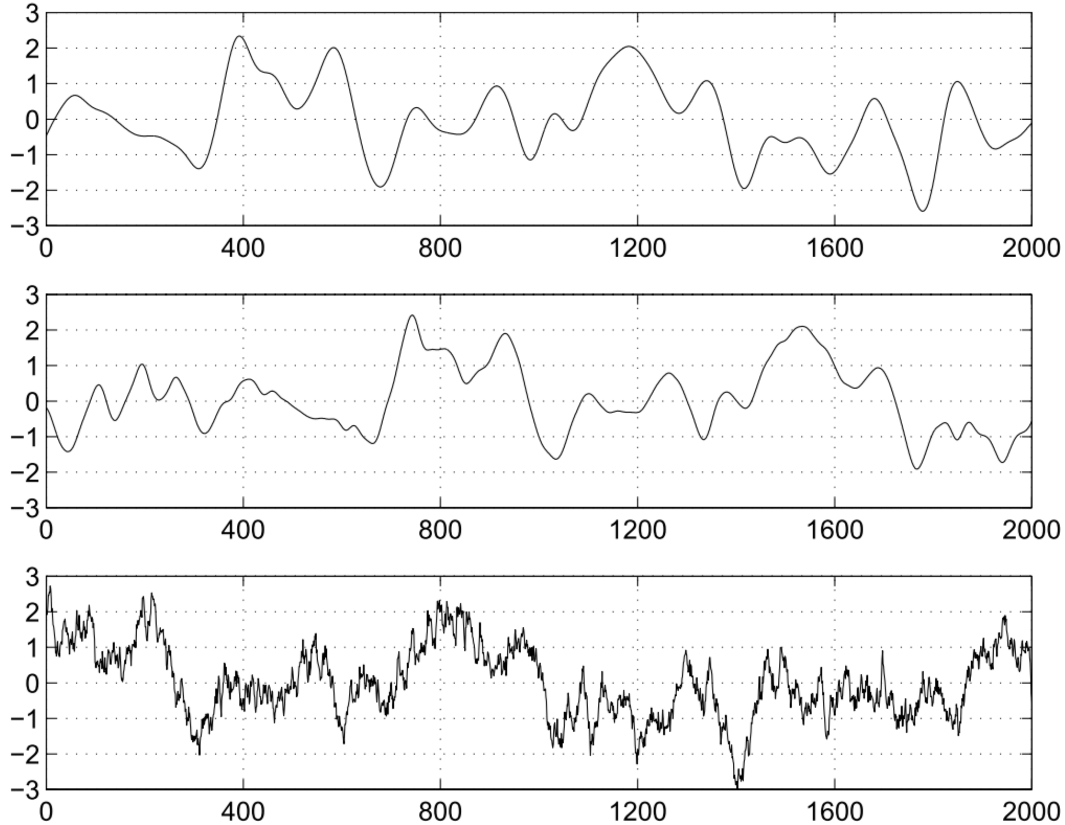
Rough interfaces: Statistical description

Generated surface – Influence of correlation type: (constant σ_h and L_c)

Gaussian:

Lorentzian:

exponential:



σ_h = 1 m
and
L_c = 50 m

Gaussian → Lorentzian → exponential: higher frequencies

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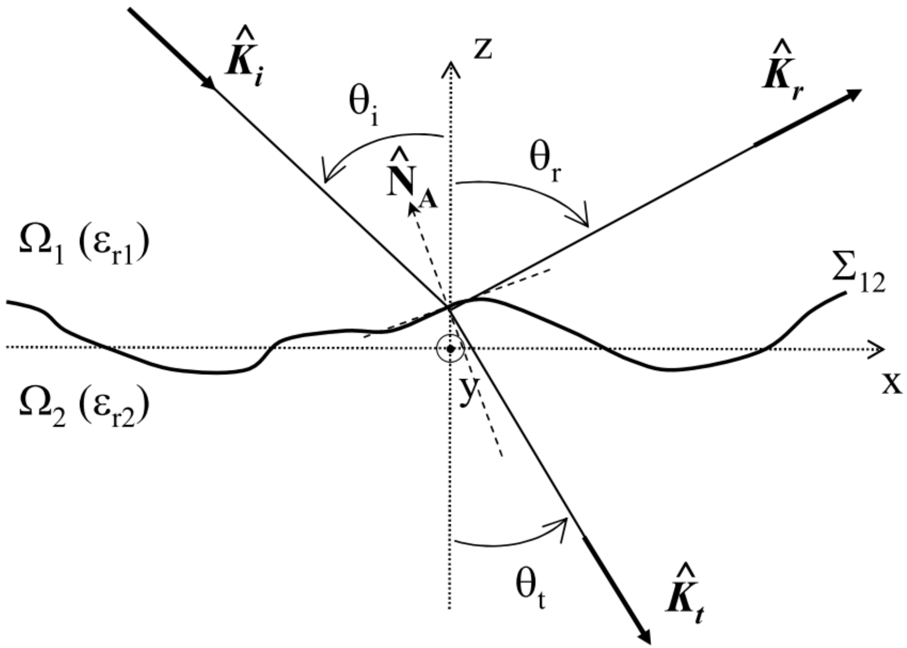
EM scattering by an interface



Incident wave $\mathbf{E_i}$ on the random rough surface $\mathbf{R} = \mathbf{R_A}$:

$$\mathbf{E_i}(\mathbf{R_A}) = E_0 \exp\left(ik_1 \widehat{\mathbf{K}}_\mathbf{i} . \widehat{\mathbf{R}}_\mathbf{A}\right) \hat{\mathbf{e}}_\mathbf{i}$$

Total field $\mathbf{E_1}$ on the random rough surface $\mathbf{R} = \mathbf{R_A}$ in the medium Ω_1:

$$\mathbf{E_1}(\mathbf{R_A}) = \mathbf{E_i}(\mathbf{R_A}) + \mathbf{E_r}(\mathbf{R_A})$$

Incident $\mathbf{E_i}$ and reflected $\mathbf{E_r}$ fields check the Helmholtz equation in Ω_1:

$$\left(\boldsymbol{\nabla}^2 + {k_1}^2\right)\mathbf{E} = \mathbf{0}$$

EM scattering by an interface

Kirchhoff-Helmholtz equations:

Equations describing the Huygens principle:

$$\mathbf{E_1} \rightarrow \mathbf{E_r} \Rightarrow \textit{Kirchhoff-Helmholtz equations}$$

- Scalar case (2D or 3D problem):

$$\forall \mathbf{R} \in \Omega_1, \;\; E_r(\mathbf{R}) = + \int_{\Sigma_A} d\Sigma_A \left(E_1(\mathbf{R_A}) \frac{\partial G_1(\mathbf{R}, \mathbf{R_A})}{\partial N_A} - G_1(\mathbf{R}, \mathbf{R_A}) \frac{\partial E_1(\mathbf{R_A})}{\partial N_A} \right)$$

with $\widehat{\mathbf{N}}_\mathbf{A}$ the normal to the surface Σ_A at considered surface point A, and $G_1(\mathbf{R_A}, \mathbf{R})$ the Green function inside Ω_1

Unknowns: Surface currents

- Vector case (3D problem):

$$\forall \mathbf{R} \in \Omega_1, \mathbf{E_r}(\mathbf{R}) = + \int_{\Sigma_A} d\Sigma_A \;\; \Big\{ i\omega\mu_0 \, \bar{G}_1(\mathbf{R}, \mathbf{R_A}) \cdot [\mathbf{N_A} \wedge \mathbf{H}_1(\mathbf{R_A})] + \boldsymbol{\nabla} \wedge \, \bar{G}_1(\mathbf{R}, \mathbf{R_A}) \cdot [\mathbf{N_A} \wedge \mathbf{E}_1(\mathbf{R_A})] \Big\}$$

EM scattering by an interface

Scattering coefficient or NRCS vs. RCS (Radar Cross Section):

- Definition of the RCS (far-field assumption):

$$RCS = 4\pi \lim_{R\to\infty} R^2 \frac{\langle |E_{d,\infty}|^2 \rangle}{|E_i|^2}$$

- Definition of the **N**RCS σ^0 (far-field assumption):

$$\sigma^0 = \lim_{R\to\infty} R^2 \frac{\langle |E_{d,\infty}|^2 \rangle}{\underbrace{L_x L_y \cos\theta_i |E_i|^2}_{\text{total incident power}}}$$

***total** incident power*

  - Coherent NRCS $\sigma^{0,coh}$: $\sigma^{0,coh} = \lim_{R\to\infty} R^2 \frac{|\langle E_{d,\infty} \rangle|^2}{L_x L_y \cos\theta_i |E_i|^2}$
  - Incoherent NRCS $\sigma^{0,inc} = \sigma^0 - \sigma^{0,coh}$:

$$\sigma^{0,inc} = \lim_{R\to\infty} R^2 \frac{\langle |E_{d,\infty}|^2 \rangle - |\langle E_{d,\infty} \rangle|^2}{L_x L_y \cos\theta_i |E_i|^2}$$

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II. EM scattering from random rough surfaces: Asymptotic models

1. **Introduction**
2. KA & SPM (2D problems)
3. Unified asymptotic models (3D problems)

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EM scattering by random rough surfaces

Random rough surfaces – Models and methods:

- **Models** of description of the EM problem:
  rigorous (“exact”) vs. *asymptotic* (approximate)
- **Methods** of resolution (computation):
  numerical (sampling) vs. *analytical* (mathematical equation)

Method / Model	**Rigorous**	**Asymptotic**
Numerical	MoM; FEM; FDTD; …	KA (KA+MSP); SPM; SSA; …
Analytical	(none)	SKA, GO; SPM; SSA; …

EM scattering by random rough surfaces

Random rough surfaces – Simple asymptotic models:

Validity domain:

physical surface-characteristic quantity (σ_h, L_c, R_c, …) compared to λ

- **"low-frequency" models** (large λ): $\lambda \gg$ physical quantity

  Example:

  SPM (Small Perturbation Method): $\sigma_h \ll a\lambda,\ a \in \mathbb{R}$

- **"high-frequency" models** (small λ): $\lambda \ll$ physical quantity

  Example:

  KA (Kirchhoff-tangent plane Approximation): $R_c \gg a\lambda,\ a \in \mathbb{R}$

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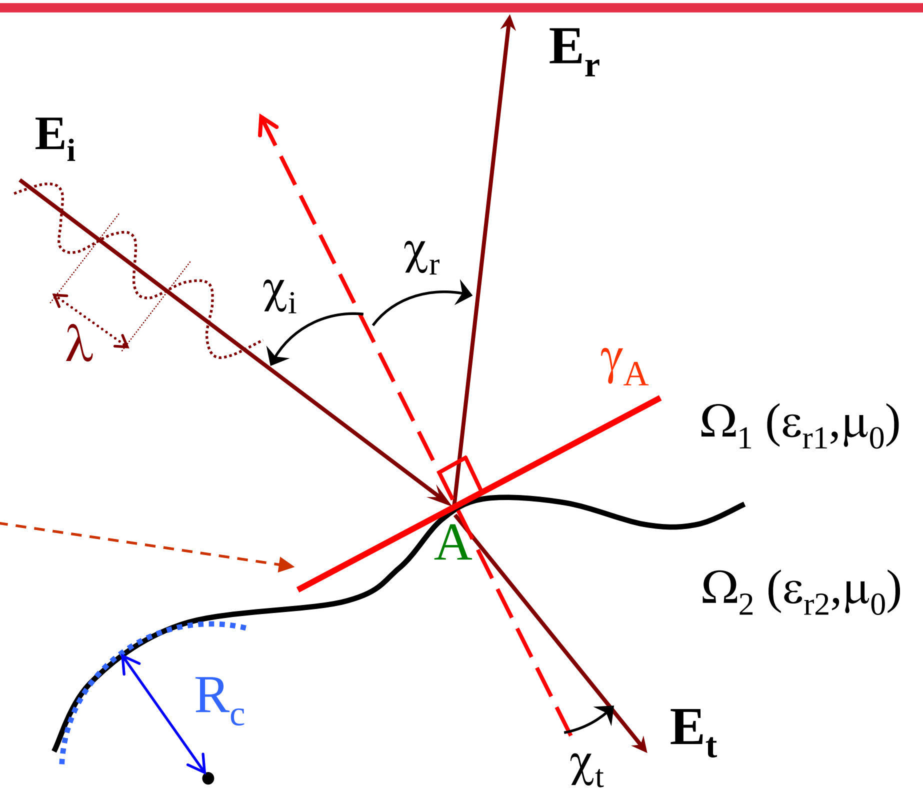
Kirchhoff Approximation (KA)

(Infinite) Tangent plane approximation:
$R_c > \lambda$

(infinite) locally flat surface



At each surface point A:
- the Snell-Descartes laws
- the Fresnel coefficients

} can be used

⇒ directions and amplitudes of $\mathbf{E_r}$, $\mathbf{E_t}$ corresponding to each scattering point A (γ_A) at any point of the considered medium

KA for 2D problems

Coherent NRCS under the KA+MSP:

General expression of the coherent NRCS $\sigma^{0,coh}$:

$$\sigma^{0,coh} = \lim_{R\to\infty} R^2 \frac{\left|\langle E_{d,\infty}\rangle\right|^2}{L_x \cos\theta_i \left|E_i\right|^2}$$

→ evaluation of coherent scattered power / $|E_0|^2$:

$$\frac{|\langle E_r^{\infty}(\mathbf{R})\rangle|^2}{2\eta_1|E_0|^2} = \frac{k_1|f_r(\mathbf{K_i},\mathbf{K_r})|^2}{4\eta_1\pi R} \left|\left\langle \int_{-L_A/2}^{+L_A/2} dx_A \; e^{i(\mathbf{K_i}-\mathbf{K_r})\cdot\mathbf{R_A}} \; \Xi(\mathbf{R_A})\right\rangle\right|^2,$$

Random variables inside $\langle\cdots\rangle$: ζ_A and $\Xi(\mathbf{R_A})$

→ Gaussian statistics:

$$\Rightarrow \quad \sigma_r^{coh}(\mathbf{K_r},\mathbf{K_i}) = \frac{1}{\cos\theta_i}\frac{2\pi}{k_1 L_A}|f_r(\mathbf{K_i},\mathbf{K_r})|^2 \mathcal{A}_r\,\delta\left(\hat{k}_{rx}-\hat{k}_{ix}\right) S_{11}^2(\mathbf{K_i},\mathbf{K_r}),$$

↑

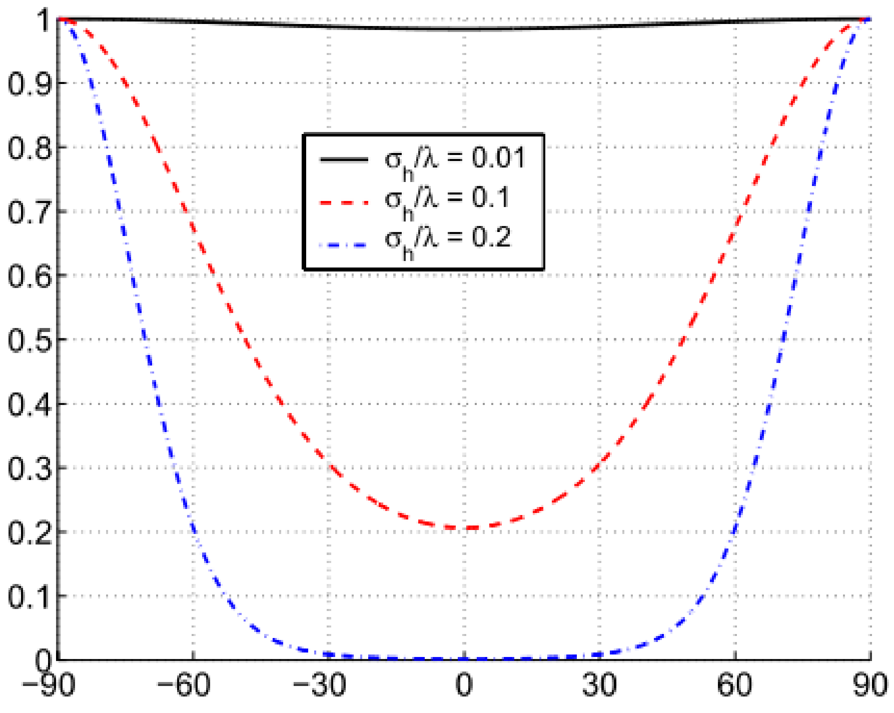
$$\mathcal{A} = e^{-4R_a{}^2} = e^{-\left(8\pi\frac{\sigma_h}{\lambda}\cos\theta_i\right)^2}$$

KA for 2D problems

Coherent NRCS under the KA+MSP:

Coherent intensity $\left|\langle E_{r,\infty}\rangle\right|^2$ in specular direction $\theta_r = \theta_i$:

→ plotting of attenuation term $\mathcal{A} = e^{-4R_a{}^2} = e^{-\left(8\pi\frac{\sigma_h}{\lambda}\cos\theta_i\right)^2}$ for $\frac{\sigma_h}{\lambda} = \{0.01; 0.1; 0.2\}$



θ_i [deg.]

- $\frac{\sigma_h}{\lambda}$ ↗ ⇒ coherent intensity ↘
- θ_i ↗ ⇒ coherent intensity ↗
  ($|\theta_i| \to 90°$ ⇒ flat EM surface)

SPM for 2D problems

Small perturbation method (2D problem)

The field scattered by the surface is:

$$E_s(\mathbf{r}) = E_{s,(0)}(\mathbf{r}) + E_{s,(1)}(\mathbf{r}) + E_{s,(2)}(\mathbf{r}) + \cdots,$$

with $E_{s,(n)}$ the n-order scattered field at $[k\zeta(x)]^n$

→ valid for $k\zeta(x) \ll 1 \;\Rightarrow\; k\sigma_h \ll 1$

- **PC surface → H (TE) polarisation (Dirichlet condition):**

Boundary condition on the surface:

$$\psi(\mathbf{r}) = \psi_i(\mathbf{r}) + \psi_s(\mathbf{r}) = 0, \;\; \mathbf{r} \in \Sigma$$

→ $\psi_{s,(0)}(\mathbf{r}) = -\psi_i(\mathbf{r}) \quad \Rightarrow \;\; \psi_{s,(1)}(\mathbf{r}) = \cdots \qquad \Rightarrow \;\; \psi_{s,(2)}(\mathbf{r}) = \cdots$

- **PC surface → V (TM) polarisation (Neumann condition):**

Boundary condition on the surface:

$$\partial\psi(\mathbf{r})/\partial n = \partial\psi_i(\mathbf{r})/\partial n + \partial\psi_s(\mathbf{r})/\partial n = 0, \;\; \mathbf{r} \in \Sigma$$

→ $\partial\psi_s(\mathbf{r})/\partial n = -\partial\psi_i(\mathbf{r})/\partial n \Rightarrow \;\; \psi_{s,(1)}(\mathbf{r}) = \cdots \qquad \Rightarrow \;\; \psi_{s,(2)}(\mathbf{r}) = \cdots$

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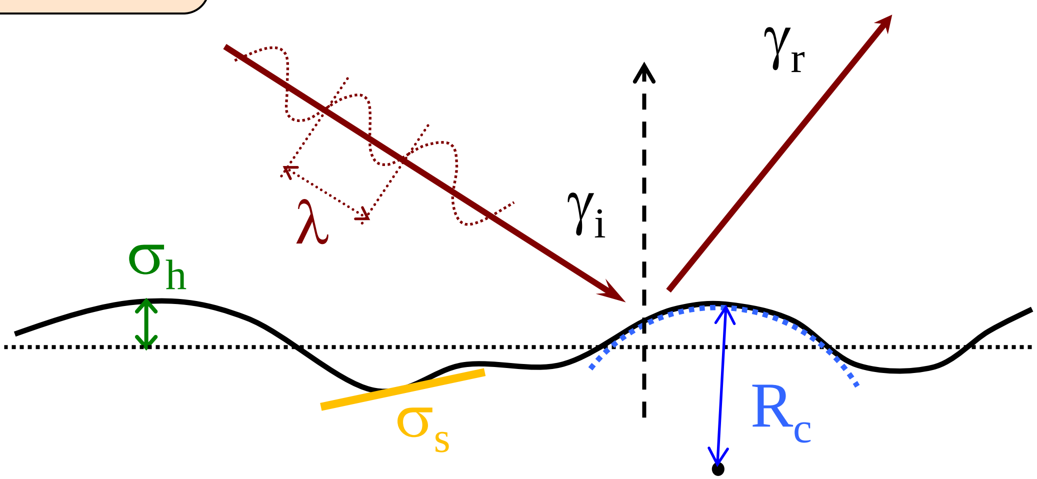
Analytic (asymptotic) methods: State of the art

Topical Review: [Elfouhaily & Guérin, WRM, 2004]

- Small Perturbation Method ($\sigma_h << \lambda$)
  …
  Low Frequency approximations
- Kirchhoff Approximation ($R_c > \lambda$)
  ↳ Geometric Optics approximation ($R_c > \lambda$ + $\sigma_h > \lambda/2$)
  ↳ Scalar Kirchhoff Approximation ($R_c > \lambda$ + $\sigma_h << \lambda$)
  High Frequency approximations

- Small Slope Approximation ($\sigma_s << |\gamma_{i,r}|$)
  …
  Unified approximations
- *etc.*

λ: incident EM wavelength
σ_h: RMS surface height
σ_s: RMS surface slope
R_c: mean surface curvature radius



Unified models for 3D problems

- Expressions of scattered field E_s and (incoherent) NRCS σ^0 / scattering amplitude (SA) $\mathbb{S}$:

$$\boldsymbol{E_s}(\boldsymbol{R}) = \int \frac{e^{j(\boldsymbol{k'}\cdot\boldsymbol{r}+q_k z)}}{q_k}\,\mathbb{S}(\boldsymbol{k'},\boldsymbol{k_0})\,d\boldsymbol{k'}\cdot\hat{\boldsymbol{E}}_0 \simeq -2j\pi\,\frac{e^{jKR}}{R}\,\mathbb{S}(\boldsymbol{k},\boldsymbol{k_0})\cdot\hat{\boldsymbol{E}}_0$$

$\Rightarrow$ Expression of SA for simple asymptotic models:

- SPM0+1+2:

$$\mathbb{S}(\boldsymbol{k},\boldsymbol{k_0}) = \underbrace{\frac{\mathbb{B}(\boldsymbol{k},\boldsymbol{k_0})}{Q_z}\,\delta(\boldsymbol{Q_H})}_{\text{SPM0}} \underbrace{-\,j\,\mathbb{B}(\boldsymbol{k},\boldsymbol{k_0})\,\hat{\eta}(\boldsymbol{Q_H})}_{\text{SPM1}} \underbrace{-\,Q_z\int_{\boldsymbol{\xi}}\mathbb{B}_2(\boldsymbol{k},\boldsymbol{k_0},\boldsymbol{\xi})\,\hat{\eta}(\boldsymbol{k}-\boldsymbol{\xi})\,\hat{\eta}(\boldsymbol{\xi}-\boldsymbol{k_0})\,d\boldsymbol{\xi}}_{\text{SPM2}}$$

- KAHF (≡ K/A+MSP):

$$\mathbb{S}(\boldsymbol{k},\boldsymbol{k_0}) = \frac{\mathbb{K}(\boldsymbol{k},\boldsymbol{k_0})}{Q_z}\int_{\boldsymbol{r}} e^{-jQ_z\eta(\boldsymbol{r})}e^{-j\boldsymbol{Q_H}\cdot\boldsymbol{r}}\,d\boldsymbol{r}$$

- SSA1+2:

$$\mathbb{S}(\boldsymbol{k},\boldsymbol{k_0}) = \underbrace{\frac{\mathbb{B}(\boldsymbol{k},\boldsymbol{k_0})}{Q_z}\int_{\boldsymbol{r}} e^{-jQ_z\eta(\boldsymbol{r})}e^{-j\boldsymbol{Q_H}\cdot\boldsymbol{r}}\,d\boldsymbol{r}}_{\text{SSA1}} \underbrace{-\,j\int_{\boldsymbol{r}}\int_{\boldsymbol{\xi}}\mathbb{M}(\boldsymbol{k},\boldsymbol{k_0};\boldsymbol{\xi})\hat{\eta}(\boldsymbol{\xi})e^{+j\boldsymbol{\xi}\cdot\boldsymbol{r}}\,d\boldsymbol{\xi}\,e^{-jQ_z\eta(\boldsymbol{r})}e^{-j\boldsymbol{Q_H}\cdot\boldsymbol{r}}\,d\boldsymbol{r}}_{\text{SSA2}}$$

→ SSA1: same structure as KAHF, but $\mathbb{B}$ kernel instead of $\mathbb{K}$ kernel

Unified models for 3D problems

Expressions of (incoherent) monostatic NRCS σ^0 $(\mathbf{k} = -\mathbf{k_0})$:

- SPM1:

$$\sigma^0_{pq}(\boldsymbol{k}, \boldsymbol{k_0}) = |\mathbb{B}_{pq}(\boldsymbol{k}, \boldsymbol{k_0})|^2 \, \tilde{W}(\boldsymbol{Q_H}) \rightarrow \begin{cases} \sigma^0_{vv}(\boldsymbol{k}, \boldsymbol{k_0}) = 16\pi k^4 \tilde{W}(-2\boldsymbol{k_0})\,(1+\sin^2\theta_0)^2, \\ \sigma^0_{vh}(\boldsymbol{k}, \boldsymbol{k_0}) = 0, \\ \sigma^0_{hv}(\boldsymbol{k}, \boldsymbol{k_0}) = 0, \\ \sigma^0_{hh}(\boldsymbol{k}, \boldsymbol{k_0}) = 16\pi k^4 \tilde{W}(-2\boldsymbol{k_0})\,\cos^4\theta_0. \end{cases}$$

- KAHF → GO:

$$\sigma^0_{pq}(\boldsymbol{k}, \boldsymbol{k_0}) = \left|\frac{\mathbb{K}_{pq}(\boldsymbol{k}, \boldsymbol{k_0})}{Q_z}\right|^2 p_s\left(\boldsymbol{\gamma} = -\frac{\boldsymbol{Q_H}}{Q_z}\right) \rightarrow \begin{cases} \sigma^0_{vv}(\boldsymbol{k}, \boldsymbol{k_0}) = \dfrac{|r_v(0)|^2}{\cos^4\theta_0}\, p_s\left(\boldsymbol{\gamma} = \tan\theta_0\right), \\ \sigma^0_{vh}(\boldsymbol{k}, \boldsymbol{k_0}) = 0, \\ \sigma^0_{hv}(\boldsymbol{k}, \boldsymbol{k_0}) = 0, \\ \sigma^0_{hh}(\boldsymbol{k}, \boldsymbol{k_0}) = \dfrac{|r_h(0)|^2}{\cos^4\theta_0}\, p_s\left(\boldsymbol{\gamma} = \tan\theta_0\right), \end{cases}$$

- SSA1:

$$\sigma_{pq}(\boldsymbol{k}, \boldsymbol{k_0}) = \frac{1}{\pi}\left|\frac{2q_k q_0}{Q_z}\,\mathbb{B}_{pq}(\boldsymbol{k}, \boldsymbol{k_0})\right|^2 \exp\left[-Q_z^2\, W(0)\right]$$

$$\int \left\{\exp\left[+Q_z^2 W(\boldsymbol{r})\right] - 1\right\} \exp\left[-j\boldsymbol{Q_H}\cdot\boldsymbol{r}\right] d\boldsymbol{r}$$

with $W(\mathbf{r})$ the surface height autocorrelation function

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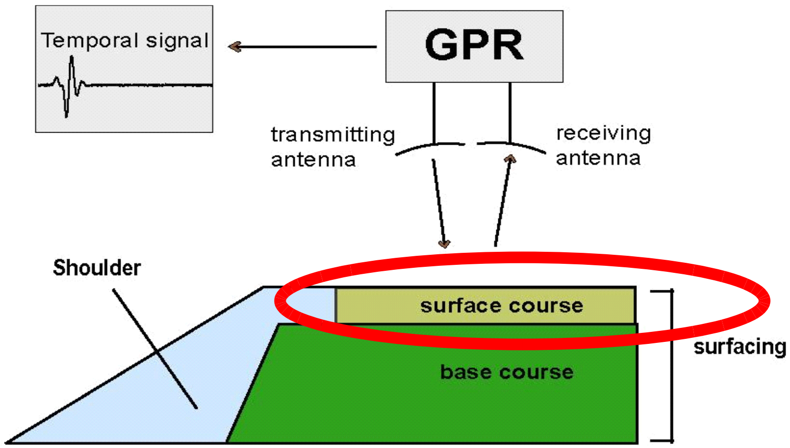
1. **Context & Objective**
2. EM modelling: Rigorous numerical method (PILE)
3. EM modelling: Analytical asymptotic method (SKA)
4. Time-domain response & Parameter estimation

Context

Pavement survey and control by NDT (Non-Destructive Testing) methods

To measure the thickness of the pavement layers

First layer of pavement: surface course (~ 5 cm)



French standards: VTAS/UTAS
(Very / Ultra Thin Asphalt Surfacing)

Tendency: reduction of the thickness
→ **Thickness: H** ~ 2-3 cm

- Radar NDT method for the pavement

Step frequency radar

GPR
(Ground Penetrating Radar)

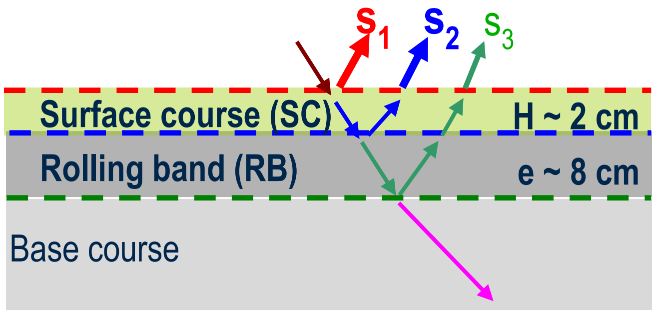
Pulse GPR

Context

General context of the study:

Electromagnetic wave scattering from *rough* thin layers in GPR context

- Better pavement thickness / medium permittivity estimation ⇒ to reduce the uncertainties
- Surface roughness estimation



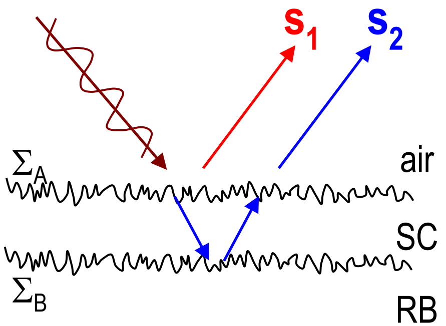
Modeling of the EM scattering of GPR from the rough thin SC of the pavement: s_1, s_2

⇒ Integration in signal processing algorithms

EM scattering modeling (*random rough* surfaces)

- one interface → air/SC interface: relatively **well-known**

- two interfaces → air/SC and SC/RB interfaces: active research



Objective

Different possible approaches:

rigorous

+ 'exact'
- long computing time
- large memory space

asymptotic

+ fast
- restricted validity domain

- frequency domain: MoM, …
- time domain: FDTD (GprMax), …

- **KA** **(Kirchhoff-tangent plane approximation)**
+ scalar approximation (SKA)
- SPM (small perturbation method)
- SSA (small slope approximation)
- …

⇒ Description of the problem to be solved (waves / surfaces & media)

Configuration of the study

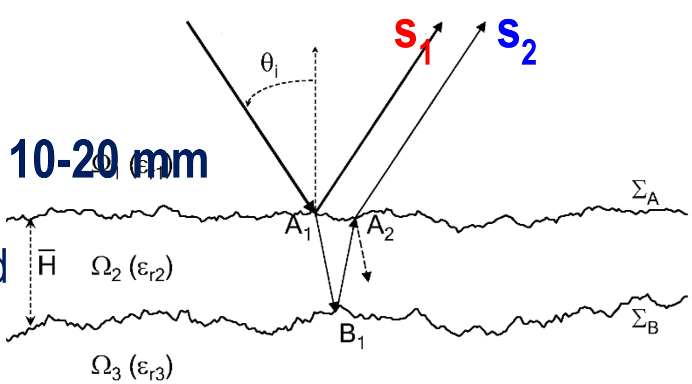
Configuration of the study (2D problems → co-polarisations)

- **Monostatic** configuration, Normal incidence (**θ_i = 0**), **Far-field** assumption
- Plane incident wave → **Gaussian beam**: Illumination width: ~ 100 mm ↔ $L_{cA} \approx$ 5-10 mm
  ⇒ *Variability of the backscattered echoes*
- Frequency study (large frequency band: B ≈ 10 GHz)

- **Homogeneous media** (OK at θ_i = 0 for this frequency range ***[Gentili and Spagnolini, TGRS, 2000]***)
- Statistical description of the rough surfaces ⇒ Realistic simulations:

• **Height PDF** $p_h(\zeta)$ (**≈Gaussian**)
  → RMS height σ_h: $\boldsymbol{\sigma_{hA} \approx 1}$ **mm**, $\boldsymbol{\sigma_{hB} \approx 2}$ **mm**

• **Height ACF** $W(x_d)$ (**≈exponential**)
  → correlation length L_c: $\boldsymbol{L_{cA} \approx}$ **5-10 mm**, $\boldsymbol{L_{cB} \approx}$ **10-20 mm**



Representation of air/SC and SC/RB surface heights

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EM modelling: Simulation parameters

Simulation parameters – PILE method (MoM-based):

Media permittivities ε_r and conductivities σ:

$$\begin{cases} \varepsilon_{r2} = 4.5 \quad - \quad \sigma_2 = 5\text{x}10^{-3}\ \text{S/m} \\ \varepsilon_{r3} = 7.0 \quad - \quad \sigma_3 = 1\text{x}10^{-2}\ \text{S/m} \end{cases}$$

Rough surfaces Σ_A and Σ_B characteristic values (σ_h and L_c):

1. σ_{hA} = **0.5** mm - L_{cA} = 6.4 mm ; σ_{hB} = **1.0** mm - L_{cB} = 15.0 mm
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Mean layer thickness H:

H = 20 mm

Radar central frequency f_0 and bandwidth B:

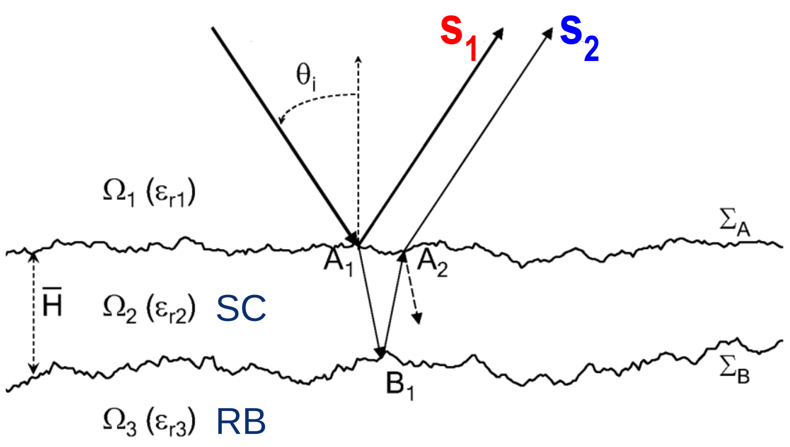
f_0 = 5.8 GHz - B = 10 GHz

Incidence angle θ_i and polarization:

θ_i = 0 deg. - V polarization

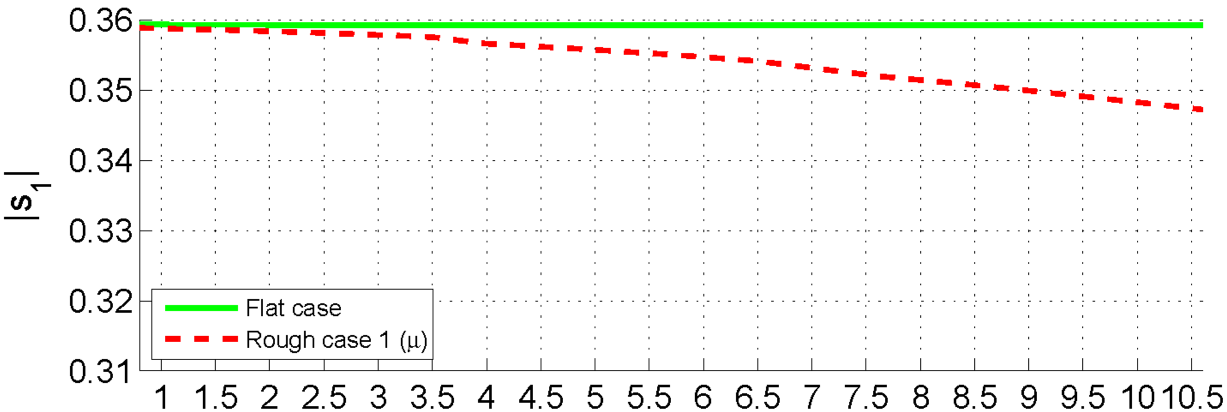
Monte-Carlo process: N = 1000 realizations

Sampling step $\Delta x = \lambda_2/8$



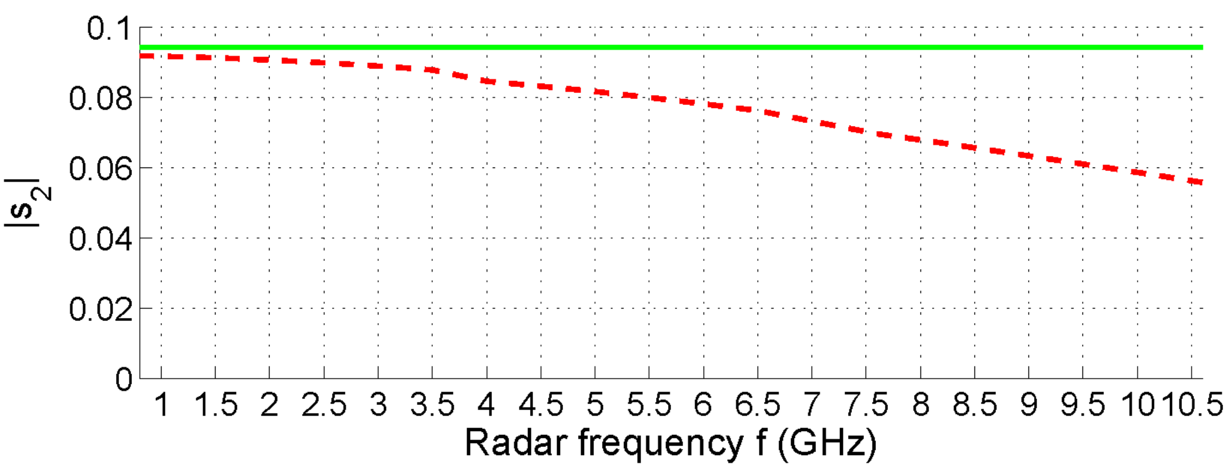
EM modelling: Numerical results (PILE)

Frequency behavior of the backscattered echoes (f ∈ [0.8; 10.8] GHz): Amplitude:



Flat case

Rough case 1 (μ)



Frequency decrease of the rough case:

Significant especially for s_2

1. σ_{hA} = **0.5** mm - L_{cA} = 6.4 mm ; σ_{hB} = **1.0** mm - L_{cB} = 15.0 mm
2. σ_{hA} = **0.5** mm - L_{cA} = 6.4 mm ; σ_{hB} = **2.0** mm - L_{cB} = 15.0 mm
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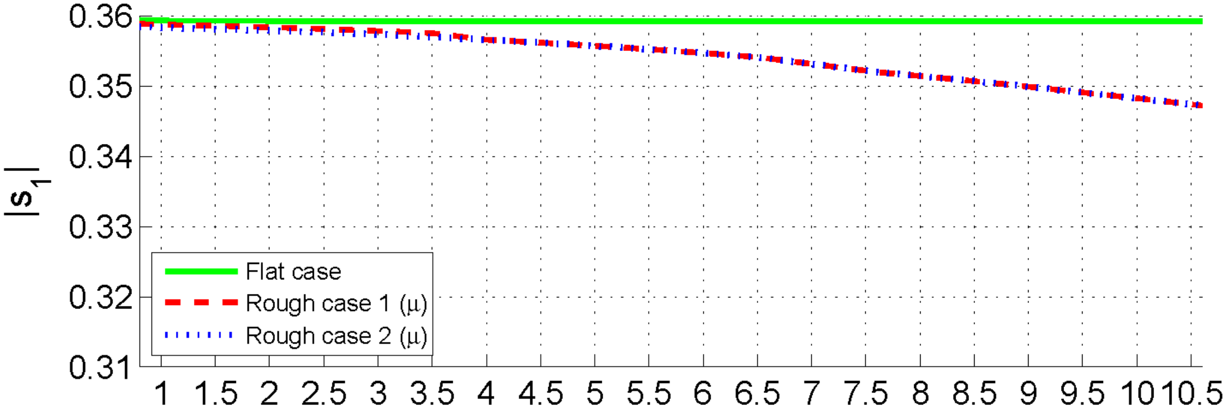
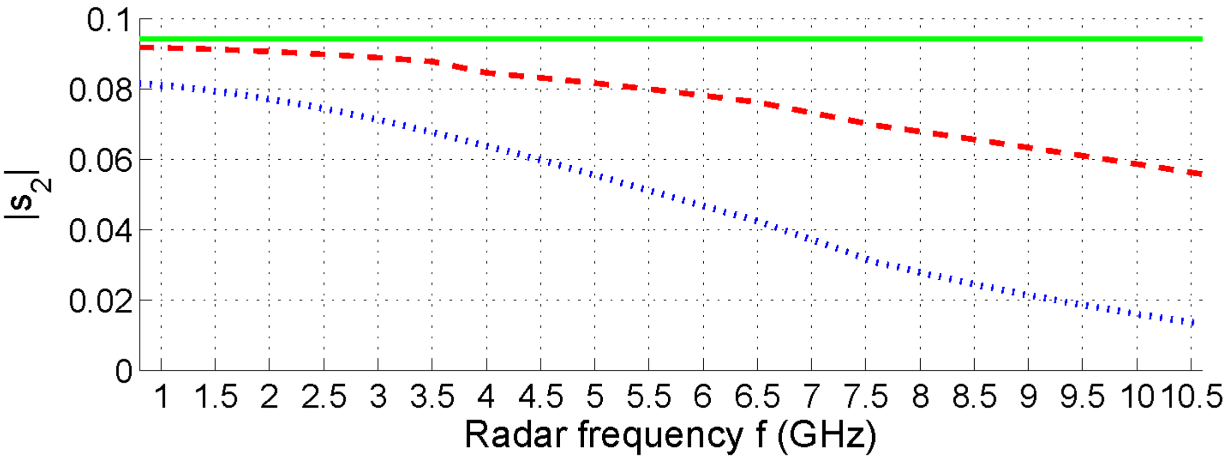
EM modelling: Numerical results (PILE)

Frequency behavior of the backscattered echoes (f ∈ [0.8; 10.8] GHz): Amplitude:



Flat case
Rough case 1 (μ)
Rough case 2 (μ)
σ_{hB} ↗



Influence of lower surface roughness σ_{hB} ↗:

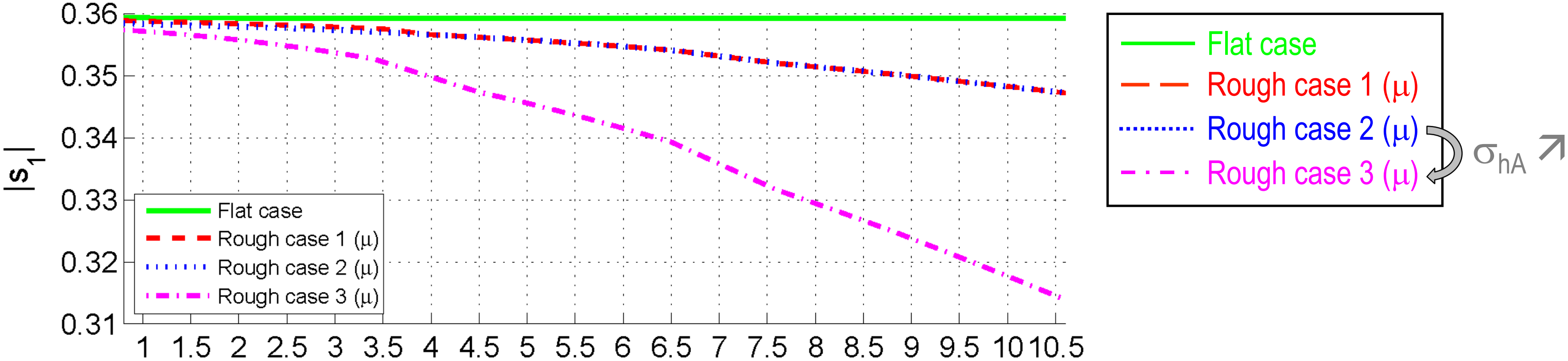
Decrease of 2nd echo amplitude $|\mathbf{s_2}|$

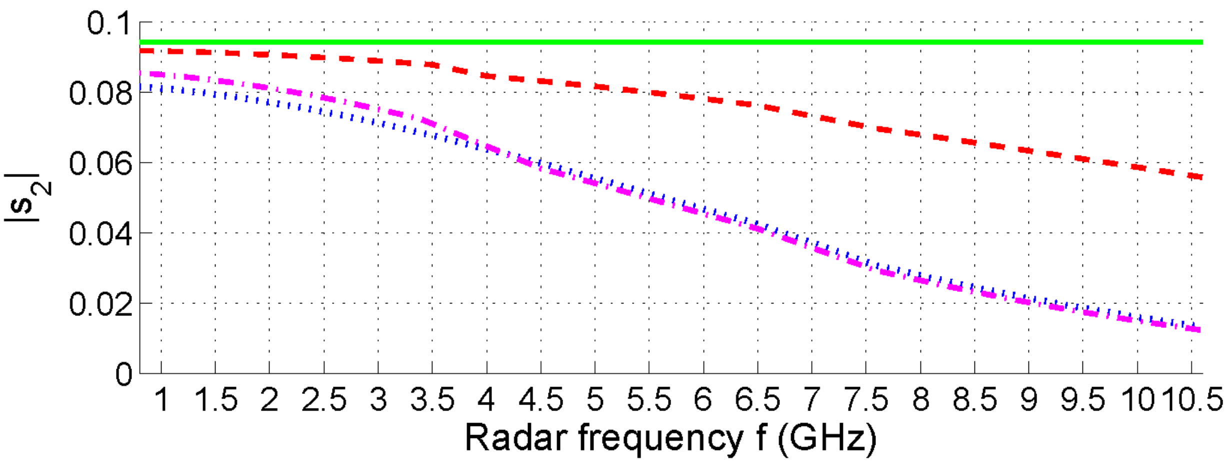
1. σ_{hA} = **0.5** mm - L_{cA} = 6.4 mm ; σ_{hB} = **1.0** mm - L_{cB} = 15.0 mm
2. σ_{hA} = **0.5** mm - L_{cA} = 6.4 mm ; σ_{hB} = **2.0** mm - L_{cB} = 15.0 mm
3. σ_{hA} = **1.0** mm - L_{cA} = 6.4 mm ; σ_{hB} = **2.0** mm - L_{cB} = 15.0 mm

σ_{hB} ↗

EM modelling: Numerical results (PILE)

Frequency behavior of the backscattered echoes ($f \in [0.8; 10.8]$ GHz): Amplitude:





Influence of upper surface roughness σ_{hA} ↗:

Decrease of 1st echo amplitude $|\mathbf{s_1}|$

Small influence on 2nd echo amplitude $|\mathbf{s_2}|$

1. σ_{hA} = **0.5** mm - L_{cA} = 6.4 mm ; σ_{hB} = **1.0** mm - L_{cB} = 15.0 mm
2. σ_{hA} = **0.5** mm - L_{cA} = 6.4 mm ; σ_{hB} = **2.0** mm - L_{cB} = 15.0 mm
3. σ_{hA} = **1.0** mm - L_{cA} = 6.4 mm ; σ_{hB} = **2.0** mm - L_{cB} = 15.0 mm

(2 → 3: σ_{hA} ↗)

OUTLINE

I. Generalities

II. EM scattering from random rough surfaces: Asymptotic models

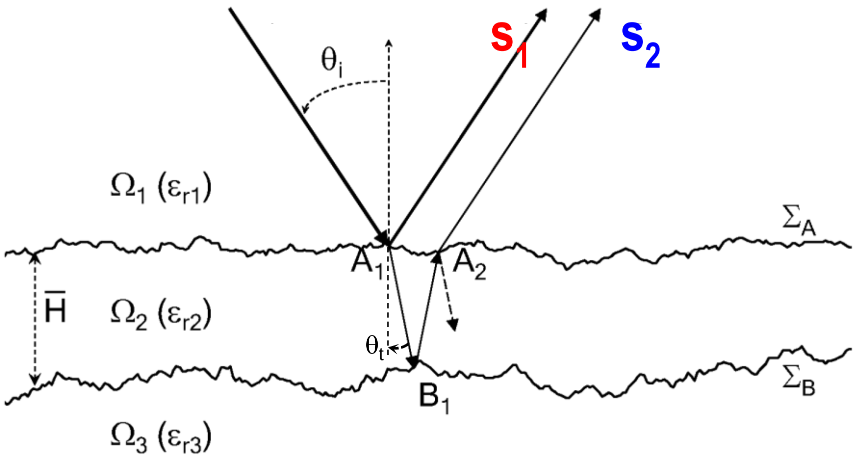
III. Applications to GPR

1. Context & Objective
2. EM modelling: Rigorous numerical method (PILE)
3. **EM modelling: Analytical asymptotic method (SKA)**
4. Time-domain response & Parameter estimation

EM modelling: Asymptotic modelling

Asymptotic computation of forward echoes s_1 and s_2:

Means: Scalar Kirchhoff-tangent plane Approximation (SKA)



Validity domain:

- Surface mean curvature radius: $\mathbf{R_c \gg \lambda}$
- Surface RMS slope: $\boldsymbol{\sigma_s \ll 1}$

Mathematical expression of first echo s_1:

$|s_{1,SKA}(f)| = |s_{1,flat}(f)| \times \exp(-2\, Ra_{r,1}^2)$,

with $Ra_{r,1} = Ra_{r12} = k_0 \sqrt{\varepsilon_{r1}}\, \sigma_{hA} \cos\theta_i$

→ "Ament model" ($Ra_{r,1}$: Rayleigh roughness parameter)

Extension to second echo s_2:

$|s_{2,SKA}(f)| = |s_{2,flat}(f)| \times \exp(-2\, Ra_{r,2}^2)$,

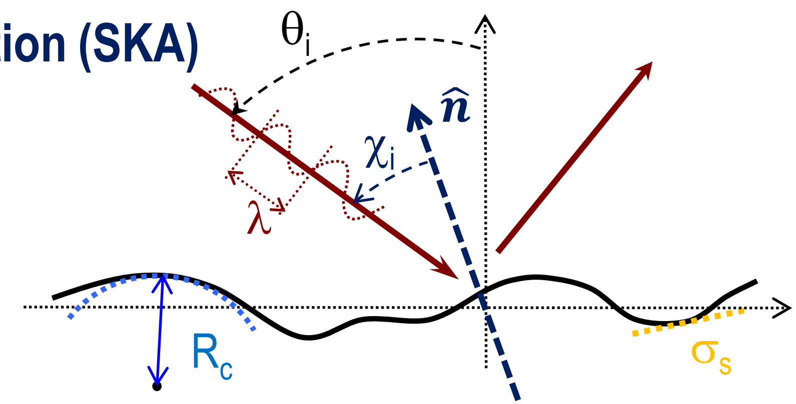
with $Ra_{r,2} = [\, 2(Ra_{t12})^2 + Ra_{r23}\,]^{1/2}$

$$\begin{cases} Ra_{t12} = k_0\, \sigma_{hA}\, |\sqrt{\varepsilon_{r1}} \cos\theta_i - \sqrt{\varepsilon_{r2}} \cos\theta_t| \,/\, 2 \\ Ra_{r23} = k_0 \sqrt{\varepsilon_{r2}}\, \sigma_{hB} \cos\theta_t \end{cases}$$

EM modelling: Asymptotic modelling

Asymptotic computation of forward scattered field – “coherent field”:

Means: Scalar Kirchhoff-tangent plane Approximation (SKA)



Validity domain:

- surface mean curvature radius: $R_c \gg \lambda$
- surface RMS slope: $\sigma_s \ll 1$

Demonstration main steps:

- Integral equations: $\forall \mathbf{R} \in \Omega_1,\ E_r(\mathbf{R}) = + \int_{\Sigma_A} d\Sigma_A \left(E_1(\mathbf{R_A}) \frac{\partial G_1(\mathbf{R}, \mathbf{R_A})}{\partial N_A} - G_1(\mathbf{R}, \mathbf{R_A}) \frac{\partial E_1(\mathbf{R_A})}{\partial N_A} \right)$
- Kirchhoff-tangent plane approximation: $E(\mathbf{R_A}) = [1 + r(\chi_i)] E_i(\mathbf{R_A})$

$$\frac{\partial E(\mathbf{R_A})}{\partial n} = i(\mathbf{K_i} \cdot \mathbf{N_A})[1 - r(\chi_i)] E_i(\mathbf{R_A})$$

- Scalar approximation: $r(\chi_i) \approx r(\theta_i)$

far-field $\Rightarrow$ $\frac{E_r^{\infty}(\mathbf{R})}{E_0} = \frac{-e^{i(k_1 R - \frac{\pi}{4})}}{\sqrt{8\pi k_1 R}} 2k_1 f_r(\mathbf{K_i}, \mathbf{K_r}) \int_{-L_A/2}^{+L_A/2} dx_A\ e^{i(\mathbf{K_i} - \mathbf{K_r}) \cdot \mathbf{R_A}}$

Coherent intensity: $L_A \to \infty \Rightarrow$ $\sigma_r^{coh}(\mathbf{K_r}, \mathbf{K_i}) = \frac{1}{\cos\theta_i} \frac{2\pi}{k_1 L_A} |f_r(\mathbf{K_i}, \mathbf{K_r})|^2 \mathcal{A}_r \delta\left(\hat{k}_{rx} - \hat{k}_{ix}\right)$

$$\mathcal{A} = |\chi_h(k_{iz} - k_{sz})|^2 \qquad \chi_h(k_{iz} - k_{sz}) \equiv \left\langle e^{i(k_{iz} - k_{sz})\zeta_A} \right\rangle$$

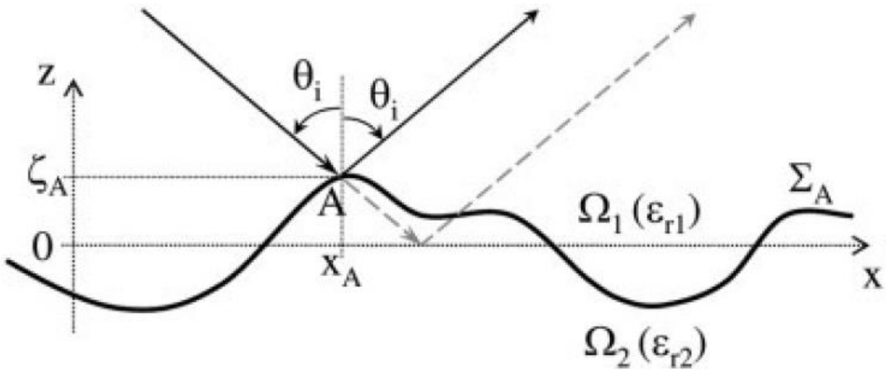
EM modelling: Asymptotic modelling

Asymptotic computation of forward scattered field – "coherent field":

<u>Means:</u> Scalar Kirchhoff-tangent plane Approximation (SKA)

<u>Validity domain:</u>

- surface mean curvature radius: $R_c >> \lambda$
- surface RMS slope: $\sigma_s << 1$



<u>Evaluation of the so-called "coherent field" in the specular (forward) direction:</u>

- starts from the evaluation of the variations of the phase of the reflected field $\delta\varphi_{r12}$:

$$\delta\varphi_{r12} = 2\, k_0 n_1\, \delta\zeta_A\, \cos\theta_i$$

- is derived after statistical average over the reflected field E_{r12}:

$$\langle E_{r12}\rangle = E_{flat}\left\langle e^{j\delta\varphi_{r12}}\right\rangle, \text{ with}$$

$$E_{flat} = r_{12}(\theta_i)\, E_{inc}$$

$$\left\langle e^{j\delta\varphi_{r12}}\right\rangle = \int_{-\infty}^{+\infty} e^{j\delta\varphi_{r12}}\, p(\zeta) d\zeta$$

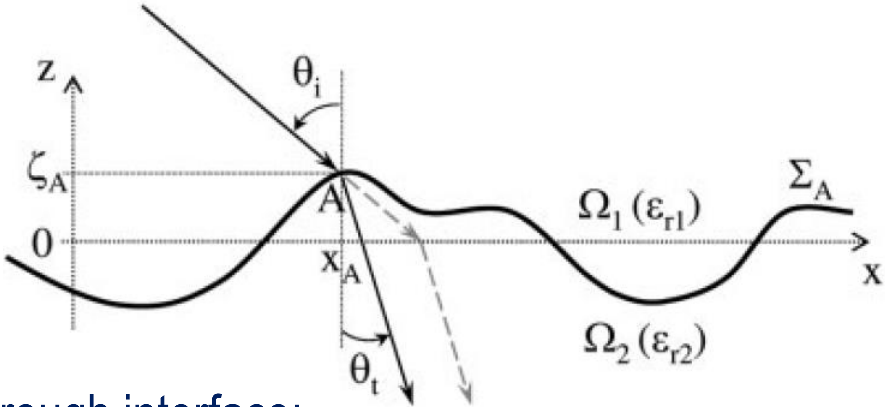
- for Gaussian statistics: $\mathcal{A}_1 = \left\langle e^{j\delta\varphi_{r12}}\right\rangle = e^{-\langle(\delta\varphi_{r12})^2\rangle/2} = e^{-2{Ra_{r12}}^2}$, with

$Ra_{r12} = k_0 n_1\, \sigma_{hA}\, \cos\theta_i$: Rayleigh roughness parameter

EM modelling: Asymptotic modelling

Asymptotic computation of forward scattered field – “coherent field”:

<u>Means:</u> Scalar Kirchhoff-tangent plane Approximation (SKA)



<u>Validity domain:</u>
- surface mean curvature radius: $R_c >> \lambda$
- surface RMS slope: $\sigma_s << 1$

<u>Extension to the transmission through a random rough interface:</u>

- starts from the evaluation of the variations of the phase of the transmitted field $\delta\varphi_{t12}$:

$$\delta\varphi_{t12} = k_0 \delta\zeta_A \,(n_1 \cos\theta_i - n_2 \cos\theta_t)$$

- is derived after statistical average over the transmitted field E_{t12}:

$$\langle E_{t12} \rangle = E_{flat} \left\langle e^{j\delta\varphi_{t12}} \right\rangle, \text{ with}$$

$$E_{flat} = t_{12}(\theta_i)\, E_{inc}$$

$$\left\langle e^{j\delta\varphi_{t12}} \right\rangle = \int_{-\infty}^{+\infty} e^{j\delta\varphi_{t12}}\, p(\zeta) d\zeta$$

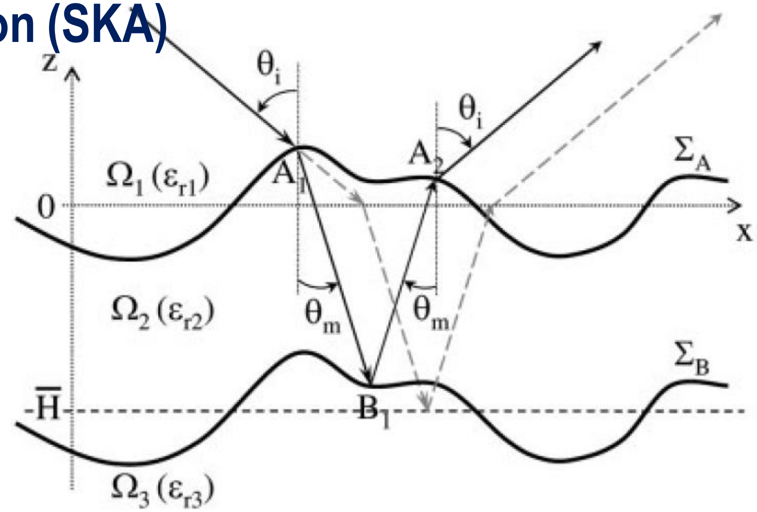
- for Gaussian statistics: $\left\langle e^{j\delta\varphi_{t12}} \right\rangle = e^{-\langle(\delta\varphi_{t12})^2\rangle/2} = e^{-2{Ra_{t12}}^2}$, with

$$Ra_{t12} = k_0 \sigma_{hA} |n_1 \cos\theta_i - n_2 \cos\theta_t| / 2$$

EM modelling: Asymptotic modelling

Asymptotic computation of forward scattered field – "coherent field":

Means: Scalar Kirchhoff-tangent plane Approximation (SKA)



Extension to the reflection from a random rough layer – second-order contribution E_2:

- variations of the phase of the reflected field $\delta\phi_2$:

$$\delta\varphi_2 = k_0 \delta\zeta_{A1} \left(n_1 \cos\theta_i - n_2 \cos\theta_m\right) + 2\, k_0 n_2\, \delta\zeta_{B1}\, \cos\theta_m + k_0 \delta\zeta_{A2} \left(n_1 \cos\theta_i - n_2 \cos\theta_m\right)$$

- mean field $\langle E_{t12}\rangle \longrightarrow$ evaluation of $\langle e^{j\delta\varphi_2}\rangle$:

Hypothesis: $\zeta_{A1}, \zeta_{B1}, \zeta_{A2}$ uncorrelated – Gaussian statistics:

$$\mathcal{A}_2 = \langle e^{j\delta\varphi_2}\rangle = e^{-\langle(\delta\varphi_2)^2\rangle/2}$$

$$\Longrightarrow \langle(\delta\varphi_2)^2\rangle/2 = 2{k_0}^2{\sigma_{hA}}^2\left(n_1\cos\theta_i - n_2\cos\theta_m\right)^2 + 4{k_0}^2{n_2}^2{\sigma_{hB}}^2\cos^2\theta_m$$

$$= 2{Ra_{t12}}^2 + {Ra_{r23}}^2$$

with

$$Ra_{r23} = k_0 n_2\, \sigma_{hB} \cos\theta_m$$

EM modelling: Asymptotic modelling

Simulation parameters: Pavement – calculation of first two echoes:

Media permittivities ε_r and conductivities σ:

$$\begin{cases} \varepsilon_{r2} = 4.5 \quad - \quad \sigma_2 = 5\text{x}10^{-3}\ \text{S/m} \\ \varepsilon_{r3} = 7.0 \quad - \quad \sigma_3 = 1\text{x}10^{-2}\ \text{S/m} \end{cases}$$

Rough surfaces Σ_A and Σ_B characteristic values (σ_h and L_c):

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Mean layer thickness H:

H = 20 mm

Radar central frequency f_0 and bandwidth B:

f_0 = 5.8 GHz - B = 10 GHz

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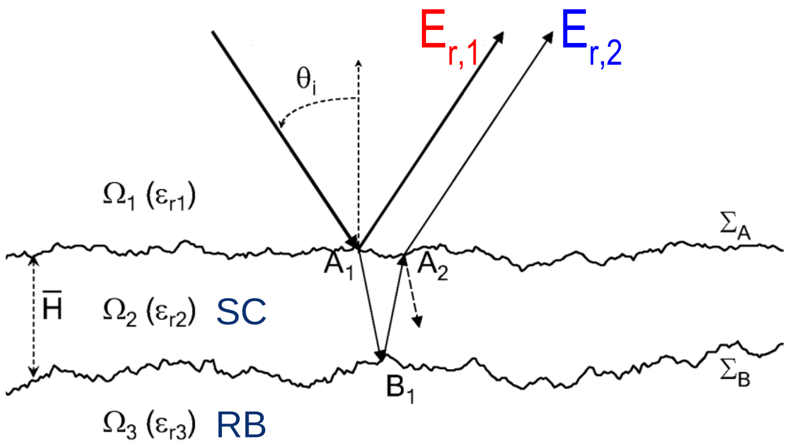
θ_i = 0 deg. - V polarization

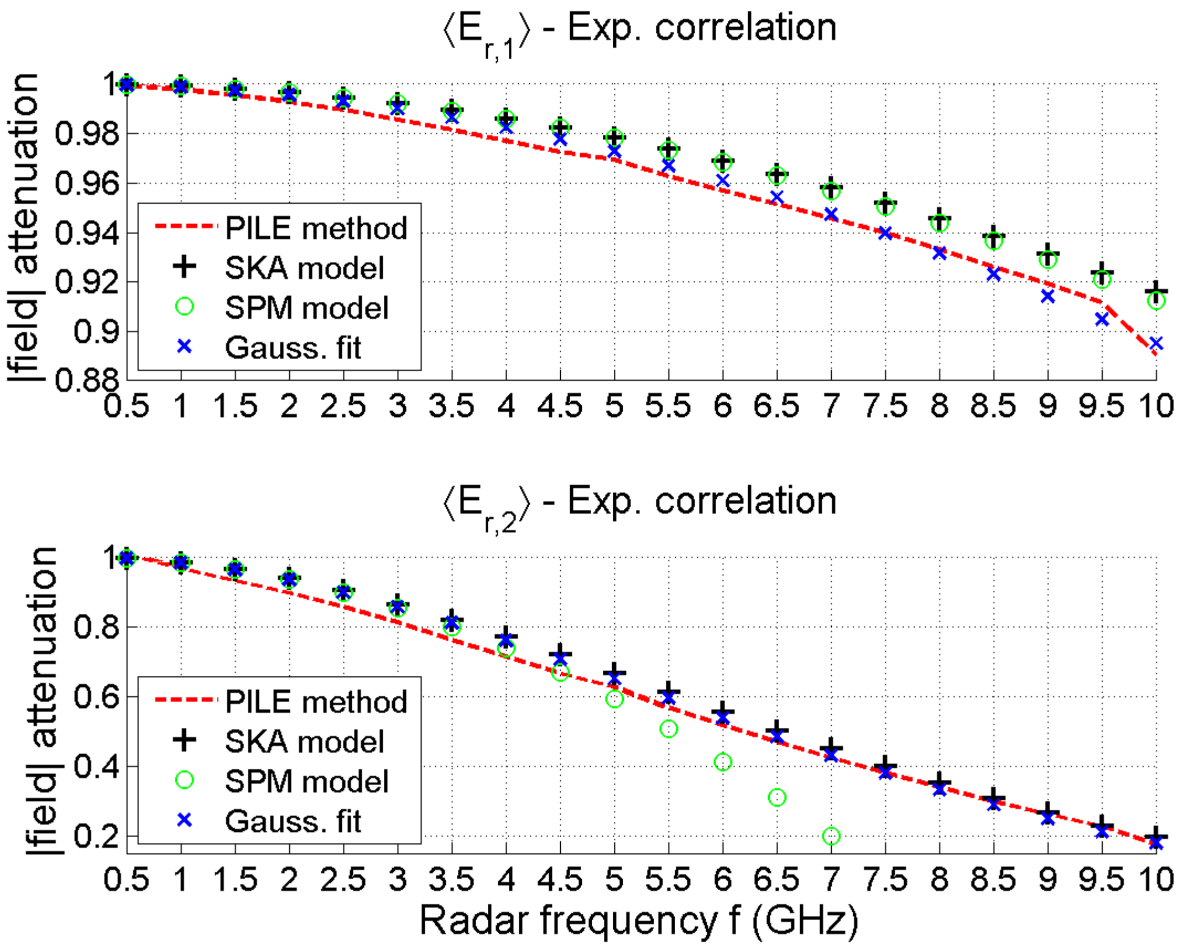
Monte-Carlo process: N = 1000 realizations

Sampling step $\Delta x = \lambda_2/8$



EM modelling: Asymptotic modelling

Frequency behavior of the backscattered echoes (f ∈ [0.8; 10.8] GHz): Amplitude:



Good agreement of SKA model

with

reference PILE (MoM) method

for both echoes $E_{r,1}$ and $E_{r,2}$

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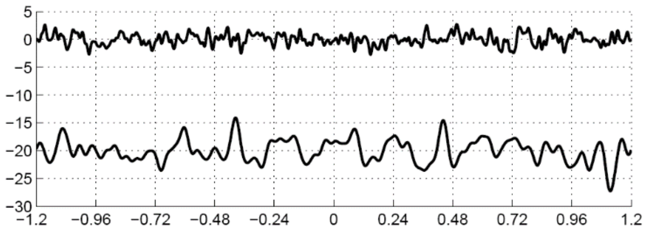
Time response to a Ricker pulse

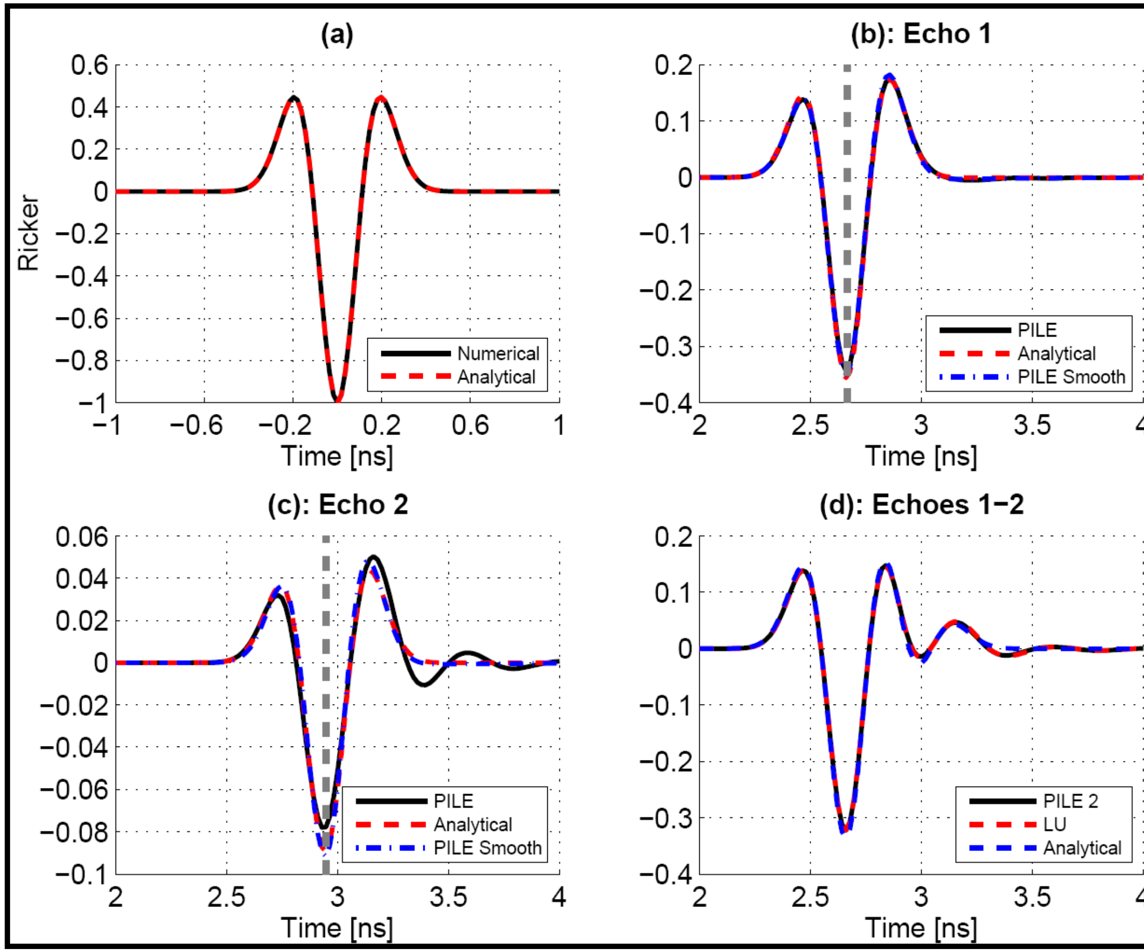


Receiver height: 40 cm

Ricker: f_c = 2 GHz and f∈[0.05; 7] GHz

LU: All echoes

PILE: First and second echoes s_1 and s_2



Echo 1: Good agreement between analytical (SKA) and PILE

Echo 2: Satisfactory agreement between analytical (SKA) and PILE

PILE 1-2 = LU ⇒ only the first two echoes contribute

EM modelling: Parameter estimation

Parameter estimation of approximate expression of echoes (exponential):

Method: Least mean squares error (LMSE)

amplitude of the k-th echo calculated from PILE (k=1,2 here)

$$R_k = \Sigma_f \left(|s_{k,data}(f)| - |s_{k,model}(f)|\right)^2$$

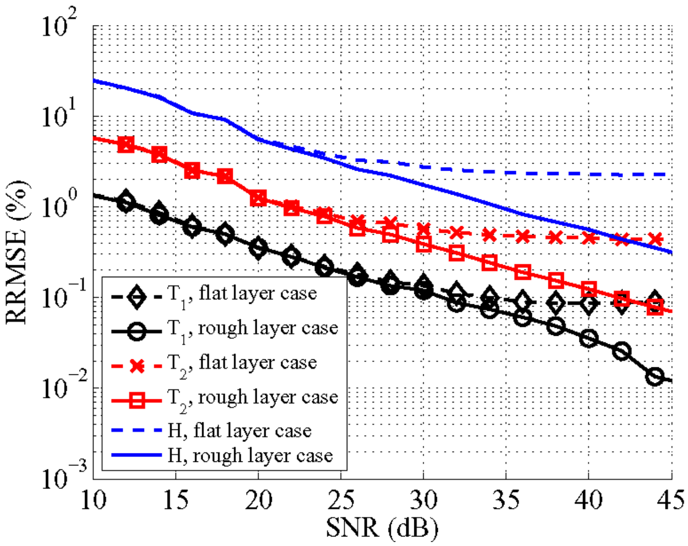
assumed expression (model):

$$|s_{k,model}(f)| = a_k \times \exp(-b_k f)$$

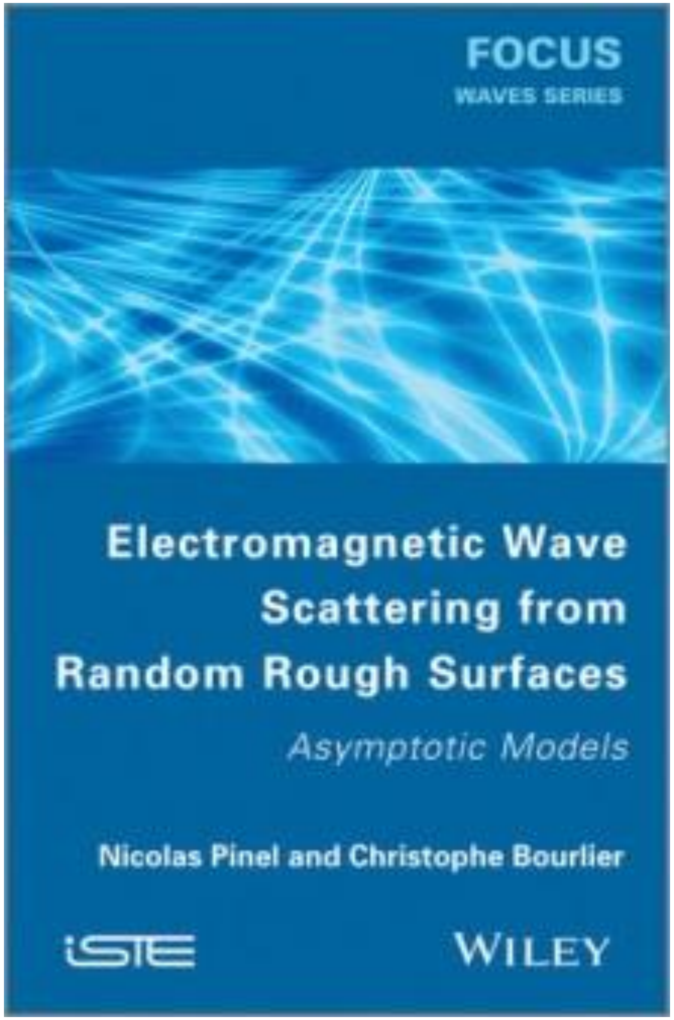
To find minimum of R_k

⇒ Determination of a_k and b_k

estimation of layer thickness H and times t_1 and t_2



Book



Electromagnetic Wave Scattering from Random Rough Surfaces – Asymptotic Models

Nicolas Pinel and Christophe Bourlier

Hardcover: 66€90 / E-book: 60€99

Short bibliography

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Questions?