# Projet SMoG - Sentinelle à Modes de Galerie

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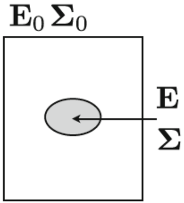
# Embedded 3D Strain Tensor Sensor Based on the Eshelby's Inclusion

# Mechanical problem

- Problematic : Measurement of strain tensor components inside a medium ($\epsilon_{ij}$)
- Eshelby's theorem : relation between the strain tensor of an ellipsoidal inclusion and the strain tensor of the medium in which is the inclusion [Eshelby, 1957]



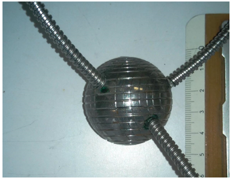

Transformation scheme, with $E_0$ the strain that corresponds to the stress $\Sigma_0$.
[François *et al.*, 2017]

The symmetrical strain tensor is :

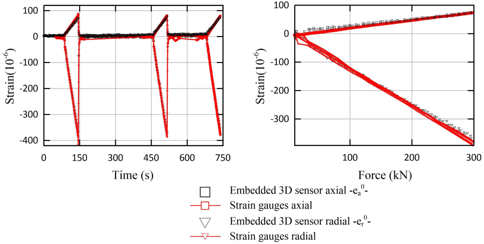
$$\begin{pmatrix} \epsilon_{11} & \epsilon_{12} & \epsilon_{13} \\ \epsilon_{12} & \epsilon_{22} & \epsilon_{23} \\ \epsilon_{13} & \epsilon_{23} & \epsilon_{33} \end{pmatrix}$$

- Six independent components $\Rightarrow$ six measurements
- What type of measurement can we make... ?

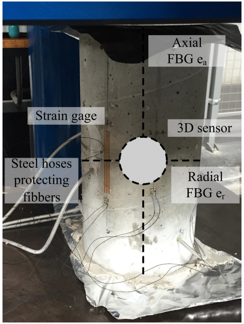
# Sensor "Sentinelle" - optical fibers



The steel version of the sensor used for concrete testing (scale in cm). [François *et al.*, 2017]



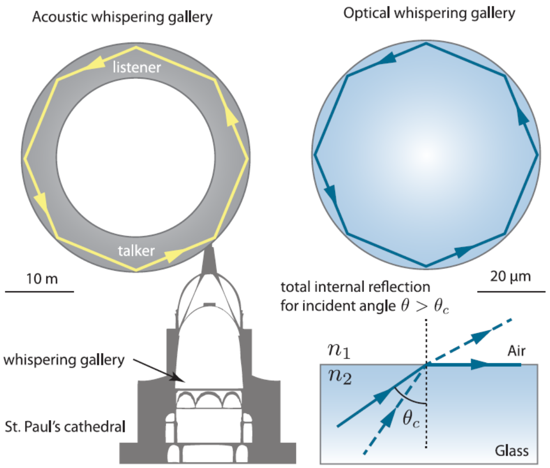

Strain versus time during compression testing on left, Strain versus force during compression testing on right. [François *et al.*, 2017]




View of the concrete specimen instrumented by 3D sensor. [François *et al.*, 2017]

- Size (cm) prevents a wider usefulness

# Sensor "SMoG" - whispering gallery modes




WGMs supported upon total internal reflection of either an acoustic (left) or an optical (right) wave. [Foreman *et al.*, 2015]

With a link between optical properties and the geometry...

$$\Delta\mathcal{L} = f(\Delta\mathcal{P})$$

with $\mathcal{L}$ the optical path and $\mathcal{P}$ the perimeter

... and a link between the geometry and the strain sensor...

$$\epsilon_{kl} = f(\Delta\mathcal{P}_i), \quad i \in 1..6, \ k \in 1..3, \ l \in 1..3$$

with $\epsilon_{kl}$ the components of the strain tensor

we can measure a strain sensor with an optical measurement :

$$\epsilon = [K]\Delta\mathcal{P}$$

# Whispering gallery modes for in-situ measurement of strain tensor

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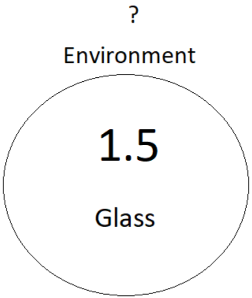
# Unknown environment

- No control over environment index...

Sensor put in concrete

- ... so no control over WGM inside the sphere



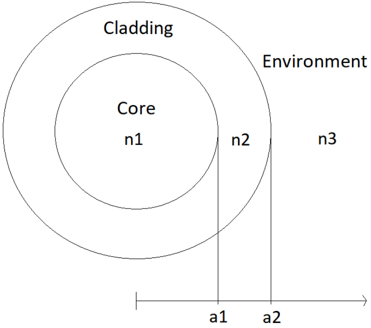

Sensor put in an unknown environment, with a unknown optical index

# A cladded sphere

- No control over environment index...

Sensor put in concrete

- ... but we can isolate the core from the environment




Studied structure

# Expression of the electromagnetic fields

# Whispering gallery mode

Electromagnetic problem for a spherical system...

$$\Delta \overrightarrow{\mathcal{F}} + n_i^2 k_0^2 \overrightarrow{\mathcal{F}} = \vec{0}$$

... but vectorial Helmholtz equation has no analytical solution in spherical coordinates !

We will then create a basis in which we will express our electromagnetical fields $\overrightarrow{\mathcal{F}}$ ($\overrightarrow{\mathcal{F}} = \vec{E}$ or $\vec{B}$), following Hansen's method [Stratton, 1941].

# Hansen's method

To create this basis, we need first to solve the scalar Helmholtz equation :

$$\Delta \mathcal{A} + n_i^2 k_0^2 \mathcal{A} = 0$$

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial}{\partial r}\mathcal{A}\right) + \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta \frac{\partial}{\partial \theta}\mathcal{A}\right) + \frac{1}{r^2 \sin^2\theta}\frac{\partial}{\partial \phi^2}\mathcal{A} + k_i^2 \mathcal{A} = 0$$

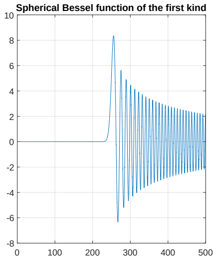
Assuming that the function $\mathcal{A}$ can be written as the product of separate variable functions...

$$\mathcal{A}(r,\theta,\phi) = \mathcal{A}_r(r)\mathcal{A}_\theta(\theta)\mathcal{A}_\phi(\phi)$$

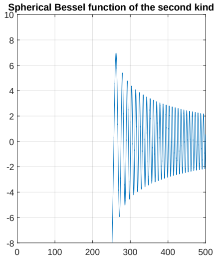
... we can find :

$$\mathcal{A}_{lm}(r,\theta,\phi) \begin{cases} \mathcal{A}_r = u_l(r) \\ \mathcal{A}_\theta = P_l^m(\cos\theta) \\ \mathcal{A}_\phi = \exp(-im\phi) \end{cases}$$

# Basis and EM fields




- Written $j_l(x) = \dfrac{\psi_\ell(x)}{x}$
- Null near 0
- Good representation of propagating fields




- Written $y_l(x) = \dfrac{\chi_\ell(x)}{x}$
- Diverges in 0
- Good representation of evanescent fields

We can build our vector basis :

$$\begin{cases} \vec{M}_{\ell m} = \vec{\nabla} \wedge \mathcal{A}_{lm}(r,\theta,\phi)\vec{e}_r \\ \vec{N}_{\ell m} = \vec{\nabla} \wedge \vec{\nabla} \wedge \mathcal{A}_{lm}(r,\theta,\phi)\vec{e}_r \end{cases}$$

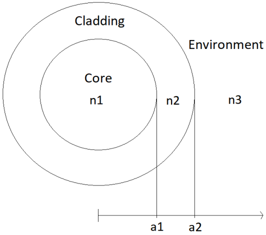
Then, electromagnetic fields in this new basis are :

$$\text{TE}: \begin{cases} \vec{E}_{\ell m} = \vec{M}_{\ell m} \\ \vec{B}_{\ell m} = -\frac{ik}{\omega}\vec{N}_{\ell m} \end{cases}$$

$$\text{TM}: \begin{cases} \vec{E}_{\ell m} = \vec{N}_{\ell m} \\ \vec{B}_{\ell m} = -\frac{ik}{\omega}\vec{M}_{\ell m} \end{cases}$$

# Dispersion equations : example of the TE case

# Continuity relations for TE case




$$\begin{cases} \text{Core} & : \mathcal{A}_r(r) = A_1 \dfrac{\psi_\ell(k_1 r)}{k_1 r} & \text{if } r < a_1 \\ \text{Cladding} & : \mathcal{A}_r(r) = A_2 \dfrac{\psi_\ell(k_2 r)}{k_2 r} + A_3 \dfrac{\chi_\ell(k_2 r)}{k_2 r} & \text{if } a_1 < r < a_2 \\ \text{Environment} & : \mathcal{A}_r(r) = A_4 \dfrac{\chi_\ell(k_3 r)}{k_3 r} & \text{if } r > a_2 \end{cases}$$

Two continuity relations for two interfaces (core/cladding and cladding/environment) :

$$\begin{pmatrix} \dfrac{\psi_\ell(k_1 a_1)}{k_1 a_1} & -\dfrac{\psi_\ell(k_2 a_1)}{k_2 a_1} & -\dfrac{\chi_\ell(k_2 a_1)}{k_2 a_1} & 0 \\ \psi'_\ell(k_1 a_1) & -\psi'_\ell(k_2 a_1) & -\chi'_\ell(k_2 a_1) & 0 \\ 0 & \dfrac{\psi_\ell(k_2 a_2)}{k_2 a_2} & \dfrac{\chi_\ell(k_2 a_2)}{k_2 a_2} & -\dfrac{\chi_\ell(k_3 a_2)}{k_3 a_2} \\ 0 & \psi_\ell(k_2 a_2) & \chi_\ell(k_2 a_2) & -\chi_\ell(k_3 a_2) \end{pmatrix} \begin{pmatrix} A_1 \\ A_2 \\ A_3 \\ A_4 \end{pmatrix} = 0$$

# Dispersion relation for TE case

We then obtain, after a couple of simplifications :

$$\begin{aligned}
&\frac{k_2}{k_3}\frac{\chi_\ell(k_3a_2)}{\chi'_\ell(k_3a_2)}[\psi_\ell(k_2a_1)\chi'_\ell(k_2a_2)-\psi'_\ell(k_2a_2)\chi_\ell(k_2a_1)]\\
&+\frac{k_2}{k_1}\frac{\psi_\ell(k_1a_1)}{\psi'_\ell(k_1a_1)}[\psi'_\ell(k_2a_1)\chi_\ell(k_2a_2)-\psi_\ell(k_2a_2)\chi'_\ell(k_2a_1)]\\
&+[\psi_\ell(k_2a_2)\chi_\ell(k_2a_1)-\psi_\ell(k_2a_1)\chi_\ell(k_2a_2)]\\
&+\frac{k_2k_2}{k_1k_3}\frac{\psi_\ell(k_1a_1)}{\psi'_\ell(k_1a_1)}\frac{\chi_\ell(k_3a_2)}{\chi'_\ell(k_3a_2)}[\psi'_\ell(k_2a_2)\chi'_\ell(k_2a_1)-\psi'_\ell(k_2a_1)\chi'_\ell(k_2a_2)]=0
\end{aligned}$$

For a null-cladding, i.e. for $k_3 \rightarrow k_2$ and $a_2 \rightarrow a_1$, we find back the classical dispersion relation for WGM in a two-medium system :

$$\frac{\chi'_l(k_2a_1)}{\chi_l(k_2a_1)}=\frac{k_1}{k_2}\frac{\psi'_l(k_1a_1)}{\psi_l(k_1a_1)}$$

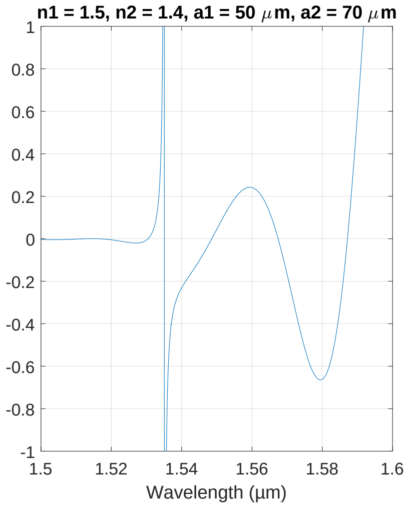
# Examples of modes

# Solving the dispersion relation in TE case...

- We will study a standard core : $a_1 = 50\ \mu$m and $n_1 = 1.5$
- We chose cladding in order to isolate the core : $a_2 = 70\ \mu$m and $n_2 = 1.4$
- A standard environment with $n_3 = 1$
- Solving the equation on $[1.5\ \mu m - 1.6\ \mu m]$ :

| | |
|---|---|
| 1512,737 | 1548,566 |
| 1516,160 | 1567,450 |
| 1530,455 | 1587,163 |

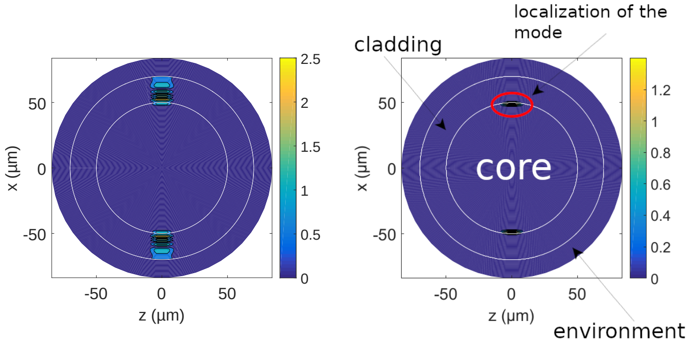
Resonance positions (in nm)




Dispersion curve for $\ell = 301$

# Map of the electric field

Drawing of the three first resonances :

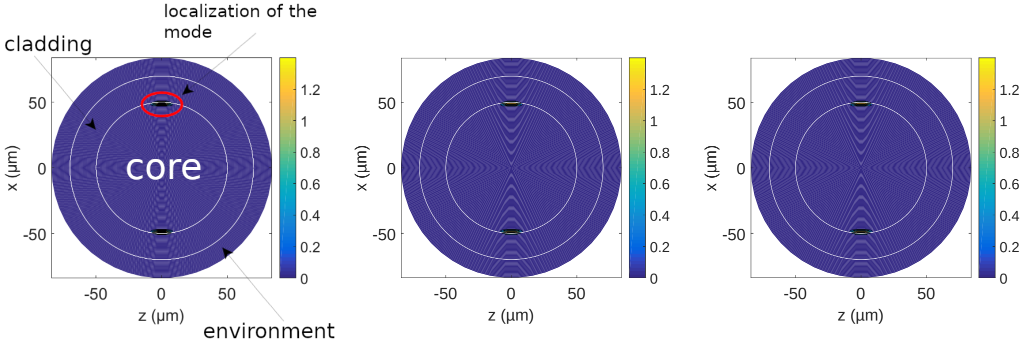



(a) 1512.737 nm (b) 1516.160 nm (c) 1530.455 nm

Electrical fields of several whispering gallery modes for $a_1 = 50\ \mu$m, $a_2 = 70\ \mu$m, $n_1 = 1.5$, $n_2 = 1.4$, $n_3 = 1$, $\ell = m = 301$

# Containment of core modes

We study the stability of the core mode 1516.160 nm if $n_3$ is changing :




(a) $n_3 = 1$ (b) $n_3 = 3$ (c) $n_3 = 5$

Electrical fields of core mode $\lambda = 1516.160$ nm for $a_1 = 50$ $\mu$m, $a_2 = 70$ $\mu$m, $n_1 = 1.5$, $n_2 = 1.4$, $\ell = m = 301$

# Conclusion

- General study of WGM in a multi-layer structure
- We establish the dispersion relation of WGM for a cladded sphere
- First numerical calculations show two types of mode : cladding modes and core modes
- Core modes seem to be insensitive to environment, they can be useful for the application of strain measurements

# Calcul numérique des modes de galerie d'une sphère élastique

GIS ECND PdL, 7 Novembre 2019, Nantes

UNIVERSITÉ DE NANTES

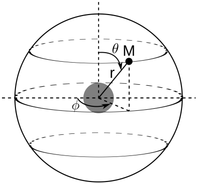
GeM Institut de Recherche en Génie Civil et Mécanique

IFSTTAR

Région PAYS DE LA LOIRE

# Contexte

Mode de galerie (optique) : mode confiné à la surface et à l'équateur. Existe-t-il des modes de galeries **élastiques** analogues ? $\equiv$ Quelles sont les résonances (modes de vibrations) d'une sphère élastique enfouie ?




Enjeux :

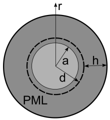
- calcul haute fréquence : ni modèle 3D, ni modèle analytique [1] (instable) $\rightarrow$ **modèle 1D semi-analytique**
- calcul des résonances d'un système ouvert : épaisseur infinie + modes à fuite (valeurs impropres croissant à l'infini [2]) $\rightarrow$ **troncature avec une Perfectly Matched Layer** (PML)

1. V. Dubrovskiy et V. Morochnik (1981), *Izv. Earth Phys* 17

2. P. Lalanne, W. Yan, K. Vynck, C. Sauvan et J.-P. Hugonin (2018), *Laser & Photonics Reviews* 12 ; M. Mansuripur, M. Kolesik et P. Jakobsen (2017), *Phys. Rev. A* 96 (1) ; M. Gallezot (2018), thèse de doct., Ecole Centrale Nantes

# Modèle numérique : forme de la solution

$$\int_{\tilde{V}} \delta\tilde{\boldsymbol{\epsilon}}^{\mathrm{T}}\tilde{\boldsymbol{\sigma}}\mathrm{d}\tilde{V} - \omega^2 \int_{\tilde{V}} \tilde{\rho}\delta\tilde{\mathbf{u}}^{\mathrm{T}}\tilde{\mathbf{u}}\mathrm{d}\tilde{V} = \int_{\tilde{V}} \delta\tilde{\mathbf{u}}^{\mathrm{T}}\tilde{\mathbf{f}}\mathrm{d}\tilde{V} + \int_{\partial\tilde{V}} \delta\tilde{\mathbf{u}}^{\mathrm{T}}\tilde{\mathbf{t}}\mathrm{d}\partial\tilde{V} \quad (1)$$




## 1. Troncature avec une PML

Atténuation le long du rayon :

$$\tilde{r}(r) = \int_0^r \gamma(\xi)\mathrm{d}\xi, \qquad (2)$$

avec

- $\gamma(r) = 1$ si $r < d$,
- $\mathbf{Im}\ \boldsymbol{\gamma}(\mathbf{r}) > \mathbf{0}$ si $d < r < d + h$.

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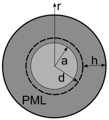


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## 2. Description angulaire analytique

Transformée en harmoniques sphériques vectorielles

$$\mathbf{u}(r,\theta,\phi) = \sum_{l\geq 0}\sum_{|m|\leq l} \mathbf{S}_l^m(\theta,\phi)\hat{\mathbf{u}}_l^m(r) \quad (4)$$

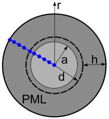
avec

$$\mathbf{S}_l^m(\theta,\phi) = \begin{bmatrix} \mathrm{Y}_l^m(\theta,\phi) & 0 & 0 \\ 0 & \frac{\partial \mathrm{Y}_l^m(\theta,\phi)}{\partial\theta} & -\frac{\partial \mathrm{Y}_l^m(\theta,\phi)}{\sin\theta\partial\phi} \\ 0 & \frac{\partial \mathrm{Y}_l^m(\theta,\phi)}{\sin\theta\partial\phi} & \frac{\partial \mathrm{Y}_l^m(\theta,\phi)}{\partial\theta} \end{bmatrix} \quad (5)$$

Harmoniques sphériques :

$$Y_l^m(\theta,\phi) = \sqrt{\frac{(2l+1)(l-m)!}{4\pi(l+m)!}} P_l^m(\cos\theta)\mathrm{e}^{\mathrm{j}m\phi} \quad (6)$$

$$\int_{\tilde{V}} \delta\tilde{\boldsymbol{\epsilon}}^{\mathrm{T}}\tilde{\boldsymbol{\sigma}}\mathrm{d}\tilde{V} - \omega^2 \int_{\tilde{V}} \tilde{\rho}\delta\tilde{\mathbf{u}}^{\mathrm{T}}\tilde{\mathbf{u}}\mathrm{d}\tilde{V} = \int_{\tilde{V}} \delta\tilde{\mathbf{u}}^{\mathrm{T}}\tilde{\mathbf{f}}\mathrm{d}\tilde{V} + \int_{\partial\tilde{V}} \delta\tilde{\mathbf{u}}^{\mathrm{T}}\tilde{\mathbf{t}}\mathrm{d}\partial\tilde{V} \quad (1)$$

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avec

- $\gamma(r) = 1$ si $r < d$,
- **Im** $\boldsymbol{\gamma(r) > 0}$ si $d < r < d + h$.

## 3. Interpolation radiale

$$\hat{\mathbf{u}}_l^{m,e}(r) = \mathbf{N}^e(r)\hat{\mathbf{U}}_l^{m,e} \quad (3)$$

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avec

$$\mathbf{S}_l^m(\theta,\phi) = \begin{bmatrix} \mathrm{Y}_l^m(\theta,\phi) & 0 & 0 \\ 0 & \frac{\partial \mathrm{Y}_l^m(\theta,\phi)}{\partial\theta} & -\frac{\partial \mathrm{Y}_l^m(\theta,\phi)}{\sin\theta\partial\phi} \\ 0 & \frac{\partial \mathrm{Y}_l^m(\theta,\phi)}{\sin\theta\partial\phi} & \frac{\partial \mathrm{Y}_l^m(\theta,\phi)}{\partial\theta} \end{bmatrix} \quad (5)$$

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# Modèle numérique : dérivation

## 4. Choix des fonctions tests

On pose $\delta \mathbf{u}^{\mathrm{T}}(r, \theta, \phi) = \delta \hat{\mathbf{u}}^{\mathrm{T}}(r) \mathbf{S}_k^{p*}$ pour bénéficier de l'orthogonalité

- des harmoniques sphériques vectorielles

$$\int_0^{\pi} \int_0^{2\pi} \mathbf{S}_k^{p*} \mathbf{S}_l^{m} \mathrm{d}\phi \sin\theta \mathrm{d}\theta = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \bar{l} & 0 \\ 0 & 0 & \bar{l} \end{bmatrix} \delta_{kl} \delta_{mp}, \tag{7}$$

  avec $\bar{l} = l(l+1)$ et $^*$ le conjugué transposé.

- des harmoniques sphériques tensorielles (voir Z. Martinec (2000), *Geophysical Journal International* 142).

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- des harmoniques sphériques vectorielles
- des harmoniques sphériques tensorielles (voir Z. Martinec (2000), *Geophysical Journal International* 142).

## Système élément fini

Après de longs calculs...on obtient :

$$\left(\mathbf{K}(l) - \omega^2 \mathbf{M}(l)\right) \hat{\mathbf{U}}_l^m = \hat{\mathbf{F}}_l^m \tag{7}$$

Matrices élémentaires :

$$\mathbf{K}_1^e(l) = \int \frac{\mathrm{d}\mathbf{N}^{e\mathrm{T}}}{\mathrm{d}r} \begin{bmatrix} C_{11} & 0 & 0 \\ 0 & \bar{l}C_{55} & 0 \\ 0 & 0 & \bar{l}C_{55} \end{bmatrix} \frac{\mathrm{d}\mathbf{N}^e}{\mathrm{d}r} \frac{\tilde{r}^2}{\gamma} \mathrm{d}r, \quad \mathbf{K}_2^e(l) = \int \frac{\mathrm{d}\mathbf{N}^{e\mathrm{T}}}{\mathrm{d}r} \begin{bmatrix} 2C_{12} & -\bar{l}C_{12} & 0 \\ \bar{l}C_{55} & -\bar{l}C_{55} & 0 \\ 0 & 0 & -\bar{l}C_{55} \end{bmatrix} \mathbf{N}^e \tilde{r} \mathrm{d}r, \tag{8}$$

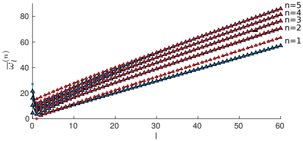
$$\mathbf{K}_3^e(l) = \int \mathbf{N}^{e\mathrm{T}} \begin{bmatrix} \bar{l}C_{55} + 4(C_{23}+C_{44}) & -\bar{l}(C_{55}+2(C_{23}+C_{44})) & 0 \\ -\bar{l}(C_{55}+2(C_{23}+C_{44})) & \bar{l}^2 C_{23} + \bar{l}C_{55} + 2\bar{l}(\bar{l}-1)C_{44} & 0 \\ 0 & 0 & \bar{l}C_{55} + \bar{l}(\bar{l}-2)C_{44} \end{bmatrix} \mathbf{N}^e \gamma \mathrm{d}r. \tag{9}$$

$$\mathbf{M}^e(l) = \int \rho \mathbf{N}^{e\mathrm{T}} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \bar{l} & 0 \\ 0 & 0 & \bar{l} \end{bmatrix} \mathbf{N}^e \tilde{r}^2 \gamma \mathrm{d}r. \tag{10}$$

Les fréquences propres $\omega_l^{(n)}$ sont solutions de :

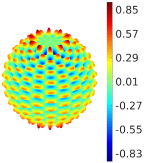
$$\left(\mathbf{K}(l) - \omega^2 \mathbf{M}(l)\right) \hat{\mathbf{U}}_l^m = \mathbf{0} \tag{11}$$

Résultats de validation pour une sphère à surface libre (sans PML). Comparaison avec les résultats de A. C. ERINGEN et E. S. ŞUHUBI (1975), t. II, Academic Press

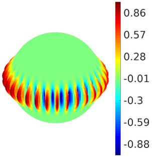



Harmonique tesserale.

**Harmonique sectorielle ≡ mode de galerie**




$l = 30,\ n = 1,\ m = 10$




$l = 30,\ n = 1,\ m = l = 30.$

Déplacement normal $u_r$ à $\overline{\omega}_{30}^{(1)} = 29.46$

## Réponse forcée par superposition modale

En utilisant l'orthogonalité *des modes propres* :

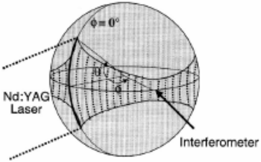
$$\mathbf{U}(\theta,\phi,t) = \sum_{l\geq 0}\sum_{|m|\leq l} \mathbf{S}_l^m(\theta,\phi)\frac{1}{2\pi}\int_{-\infty}^{+\infty}\left[\sum_{n=1}^{N}\frac{\hat{\mathbf{U}}_l^{(n)\mathrm{T}}\hat{\mathbf{F}}_l^m(\omega)\hat{\mathbf{U}}_l^{(n)}}{\omega_l^{(n)2}-\omega^2}\right]\mathrm{e}^{-\mathrm{j}\omega t}\mathrm{d}\omega. \qquad (12)$$

En rouge, la Fonction de Réponse Fréquentielle (FRF) pour une paire $(l, m)$.

Il est possible d'obtenir une onde de Rayleigh collimatée (D. Clorennec et D. Royer (2004), *Applied physics letters* 85) :

Angle de collimation

$$\theta_{\mathrm{COL}} = \sqrt{\frac{\pi c_R}{4 a f_c}} \qquad (13)$$




- Phénomène simulé correctement par le modèle (voir vidéos)
- Analyse modale : **l'onde de Rayleigh collimatée est issue de la superposition de modes propres de galerie**.

# Perspectives

- étudier l'existence de modes de galerie peu atténués (facteur de qualité élevé) en présence d'enrobant (modèle avec PML) $\rightarrow$ existence d'une onde piégée à l'interface ?
- le cas échéant, étudier les conditions de génération de ces modes (type de source à utiliser)

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Merci pour votre attention !